Exponents, Radicals, and Polynomials

Unit 4

Unit Overview

In this unit you will explore multiplicative patterns and representations of nonlinear data. Exponential growth and decay will be the basis for studying exponential functions. You will investigate the properties of powers and radical expressions. You will also perform operations with radical and rational expressions.

Unit 4 Vocabulary

Add these words and others you encounter in this unit to your vocabulary notebook.

- coefficient
- degree of a polynomial
- difference of two squares
- factor
- polynomial
- radical expression
- rational expression
- term

Essential Questions

? How do multiplicative patterns model the physical world?

? How are adding and multiplying polynomial expressions different from each other?

EMBEDDED ASSESSMENTS

This unit has two embedded assessments, following Activities 4.3 and 4.8. They will give you an opportunity to demonstrate what you have learned.

Embedded Assessment 1

Exponential Functions   p. 229

Embedded Assessment 2

Polynomial Operations
and Factoring   p. 273

Write your answers on notebook paper or grid paper. Show your work.

1. Find the greatest common factor of 36 and 54.

2. Give the prime factorization of 90.

3. Which of the following is equivalent to $39 \cdot 26 + 39 \cdot 13$?

   a. 13^9 **b.** $13^4 \cdot 14$

   c. $13^2 \cdot 3^2 \cdot 2$ **d.** $13^2 \cdot 3^2$

4. Explain 2 ways to evaluate $15(90 - 3)$.

5. Identify the coefficient, base and exponent of $4x^5$.

6. Complete the following table to create a linear relationship.

x	2	4	6	8	10
y	3	5			

7. Graph the function described in the table in question 6.

8. Use ratios to model the following:

   a. 7.5

   b. Caleb receives 341 of the 436 votes cast for class president.

   c. Students in Mr. Bulluck's Class

Girls	Boys
12	19

   1. girls to boys

   2. boys to total class members

9. For each of the following, tell which describes the number:
   I. An integer
   II. A rational number
   III. An irrational number

   a. $\sqrt{25}$ **b.** $\frac{4}{3}$ **c.** 2.16 **d.** π

10. Calculate.

    a. $\frac{1}{2} + \frac{3}{8}$ **b.** $\frac{5}{12} - \frac{1}{3}$

   **c.** $\frac{3}{4} \cdot \frac{2}{5}$ **d.** $\frac{5}{8} \div \frac{3}{4}$

SUGGESTED LEARNING STRATEGIES: Marking the Text, Group Discussion, Create Representations, Predict and Confirm

An *iceberg* is a large piece of freshwater ice that has broken off from a glacier or ice shelf and is floating in open sea water. Icebergs are classified by size. The smallest sized iceberg is called a "growler".

A growler was found floating in the ocean just off the shore of Greenland. Its volume above water was approximately 27 cubic meters.
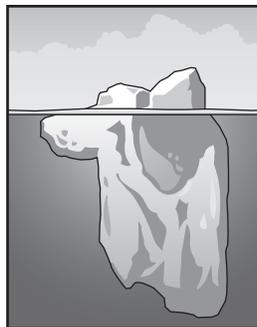
1. Two icebergs float near this growler. One iceberg's volume is 3^4 times greater than the growler. The second iceberg's volume is 2^8 times greater than the growler. Which iceberg has the larger volume? Explain below.

2. What is the meaning of 3^4 and 2^8? Why do you think exponents are used when writing numbers?

3. Suppose the original growler's volume under the water is 9 times the volume above. How much of its ice is below the surface?

4. Write your solution to Item 3 using powers. Complete the equation below. Write the missing terms as a power of 3.

volume above water $\cdot \, 3^2 =$ volume below the surface

$$\boxed{} \cdot 3^2 = \boxed{}$$

5. Look at the equation you completed for Item 4. What relationship do you notice between the exponents on the left side of the equation and the exponent on the right?

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Predict and Confirm, Think/Pair/Share, Create Representations, Note Taking, Group Discussion

6. Use the table below to help verify the pattern you noticed in Item 5. First write each product in the table in expanded form. Then express the product as a single power of the given base. The first one has been done for you.

Original Product	Expanded Form	Single Power
$2^2 \cdot 2^4$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6
$5^3 \cdot 5^2$		
$x^4 \cdot x^7$		
$a^6 \cdot a^2$		

7. Based on the pattern you observed in the table in Item 6, write the missing exponent in the box below to complete the **Product of Powers Property** for exponents.

$$a^m \cdot a^n = a^{\boxed{}}$$

8. The density of an iceberg is determined by dividing its mass by its volume. Suppose a growler had a mass of 59,049 kg and a volume of 81 cubic meters. Compute the density of the iceberg.

9. Write your solution to Item 8 using powers of 9.

$$\frac{\boxed{\textbf{Mass}}}{\boxed{\textbf{Volume}}} = \boxed{\textbf{Density}}$$

10. What pattern do you notice in the equation you completed for Item 9?

CONNECT TO SCIENCE

The formula for density is
$$D = \frac{M}{V}$$
where D is density, M is mass, and V is volume.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Note Taking, Predict and Confirm

My Notes

11. Use the table to help verify the patterns you noticed in Item 9. First write each quotient in the table below in expanded form. Then express the quotient as a single power of the given base. The first one has been done for you.

Original Product	Expanded Form	Single Power
$\dfrac{2^5}{2^2}$	$\dfrac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = \dfrac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2}}$	2^3
$\dfrac{5^8}{5^6}$		
$\dfrac{a^3}{a^1}$		
$\dfrac{x^7}{x^3}$		

12. Based on the pattern you observed in Item 11, write the missing exponent in the box below to complete the **Quotient of Powers Property** for exponents.

$$\frac{a^m}{a^n} = a^{\boxed{}}, \text{ where } a \neq 0$$

The product and quotient properties of exponents can be used to simplify expressions.

EXAMPLE 1

Simplify: $2x^5 \cdot 5x^4$

Step 1: *Group powers with the same base.* $2x^5 \cdot 5x^4 = 2 \cdot 5 \cdot x^5 \cdot x^4$

Step 2: *Product of Powers Property* $\qquad = 10x^{5+4}$

Step 3: *Simplify the exponent.* $\qquad = 10x^9$

Solution: $2x^5 \cdot 5x^4 = 10x^9$

My Notes

EXAMPLE 2

Simplify $\dfrac{2x^5y^4}{xy^2}$.

Step 1: Group powers with the same base. $\dfrac{2x^5y^4}{xy^2} = 2 \cdot \dfrac{x^5}{x} \cdot \dfrac{y^4}{y^2}$

Step 2: Quotient of Powers Property $= 2x^{5-1} \cdot y^{4-2}$

Step 3: Simplify the exponent. $= 2x^4y^2$

Solution: $\dfrac{2x^5y^4}{xy^2} = 2x^4y^2$

TRY THESE A

Simplify each expression.

a. $(4xy^4)(-2x^2y^5)$ **b.** $\dfrac{2a^2b^5c}{4ab^2c}$ **c.** $\dfrac{6y^3}{18x} \cdot 2xy$

13. Write each quotient in expanded form and simplify it. Then apply the quotient property of exponents. The first one has been done for you.

Original Quotient	Expanded Form	Single Power
$\dfrac{2^5}{2^8}$	$\dfrac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} =$ $\dfrac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2} = \dfrac{1}{2^3}$	$2^{5-8} = 2^{-3}$
$\dfrac{5^3}{5^6}$		
$\dfrac{a^3}{a^8}$		
$\dfrac{x^4}{x^{10}}$		

SUGGESTED LEARNING STRATEGIES: Note Taking, Activating Prior Knowledge, Predict and Confirm, Create Representations, Look for a Pattern

14. Based on the pattern you observed in Item 13, write the missing exponent in the box below to complete the **Negative Power Property** for exponents.

$$\frac{1}{a^n} = a^{\boxed{}}, \text{ where } a \neq 0$$

15. Write each quotient in expanded form and simplify it. Then apply the quotient property of exponents. The first one has been done for you.

Original Quotient	Expanded Form	Single Power
$\dfrac{2^4}{2^4}$	$\dfrac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = \dfrac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 1$	$2^{4-4} = 2^0$
$\dfrac{5^6}{5^6}$		
$\dfrac{a^3}{a^3}$		

16. Based on the pattern you observed in Item 15, fill in the box below to complete the **Zero Power Property** of exponents.

$$a^0 = \boxed{}, \text{ where } a \neq 0$$

You can use the negative power property and the zero power property of exponents to evaluate and simplify expressions.

TRY THESE **B**

Simplify each expression.

a. 2^{-3}　　**b.** $\dfrac{10^2}{10^{-2}}$　　**c.** $3^{-2} \cdot 5^0$　　**d.** $(-3.75)^0$

My Notes

CONNECT TO AP

In calculus, an expression containing a negative exponent often is preferable to one written as a quotient.

My Notes

When evaluating and simplifying expressions, you can apply the properties of exponents and then write the answer without negative or zero powers.

EXAMPLE 3

Simplify $5x^{-2}yz^0 \cdot \dfrac{3x^4}{y^4}$ and write without negative powers.

Step 1: Commutative Property $5x^{-2}yz^0 \cdot \dfrac{3x^4}{y^4}$
$$= 5 \cdot 3 \cdot x^{-2} \cdot x^4 \cdot y^1 \cdot y^{-4} \cdot z^0$$

Step 2: Apply the exponent rules. $\quad = 5 \cdot 3 \cdot x^{-2+4} \cdot y^{1-4} \cdot z^0$

Step 3: Simplify the exponents. $\quad = 15 \cdot x^2 \cdot y^{-3} \cdot 1$

Step 4: Write without negative exponents. $\quad = \dfrac{15x^2}{y^3}$

Solution: $\quad 5x^{-2}yz^0 \cdot \dfrac{3x^4}{y^4} = \dfrac{15x^2}{y^3}$

TRY THESE C

Simplify and write without negative powers.

a. $2a^2b^{-3} \cdot 5ab$ **b.** $\dfrac{10x^2y^{-4}}{5x^{-3}y^{-1}}$ **c.** $(-3xy^{-5})^0$

17. Write each expression in expanded form. Then write the expression using a single exponent with the given base. The first one has been done for you.

Original Expression	Expanded Form	Single Exponent
$(2^2)^4$	$2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 =$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^8
$(5^5)^3$		
$(x^3)^4$		

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations, Note Taking, Predict and Confirm, Look for a Pattern

My Notes

18. Based on the pattern you observed in Item 17, write the missing exponent in the box below to complete the **Power of a Power Property** for exponents.

$$(a^m)^n = a^{\boxed{}}$$

19. Write each expression in expanded form and group like terms. Then write the expression as a product of powers. The first one has been done for you.

Original Expression	Expanded Form	Product of Powers
$(2x)^4$	$2x \cdot 2x \cdot 2x \cdot 2x =$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x$	$2^4 x^4$
$(-4a)^3$		
$(x^3 y^2)^4$		

20. Based on the pattern you observed in Item 19, write the missing exponents in the boxes below to complete the **Power of a Product Property** for exponents.

$$(ab)^m = a^{\boxed{}} \cdot b^{\boxed{}}$$

21. Use the patterns you have seen. Predict and write the missing exponents in the boxes below to complete the **Power of a Quotient Property** for exponents.

$$\left(\frac{a}{b}\right)^m = \frac{a^{\boxed{}}}{b^{\boxed{}}}, \text{ where } b \neq 0$$

MATH TIP

Use a graphic organizer to record the properties of exponents you learned in this activity.

My Notes

You can apply these power properties and the exponent rules you have already learned to simplify expressions.

EXAMPLE 4

Simplify $(2x^2y^5)^3(3x^2)^{-2}$ and write without negative powers.

Step 1: *Power of a Power Property*

$$(2x^2y^5)^3(3x^2)^{-2}$$
$$= 2^3 x^{2 \cdot 3} y^{5 \cdot 3} \cdot 3^{-2} \cdot x^{2 \cdot -2}$$

Step 2: *Simplify the exponents and the numerical terms.*

$$= 8 \cdot x^6 y^{15} \cdot \frac{1}{3^2} \cdot x^{-4}$$

Step 3: *Commutative Property*

$$= 8 \cdot \frac{1}{9} x^6 \cdot x^{-4} y^{15}$$

Step 4: *Product of Powers Property*

$$= \frac{8}{9} x^{6-4} y^{15}$$

Step 5: *Simplify the exponents.*

$$= \frac{8}{9} x^2 y^{15}$$

Solution: $(2x^2y^5)^3(3x^2)^{-2} = \frac{8}{9} x^2 y^{15}$

EXAMPLE 5

Simplify $\left(\dfrac{x^2 y^{-3}}{z}\right)^2$.

Step 1: *Power of a Quotient Property*

$$\left(\frac{x^2 y^{-3}}{z}\right)^2 = \frac{x^{2 \cdot 2} y^{-3 \cdot 2}}{z^2}$$

Step 2: *Simplify the exponents.*

$$= \frac{x^4 y^{-6}}{z^2}$$

Step 3: *Negative Exponents Property*

$$= \frac{x^4}{y^6 z^2}$$

Solution: $\left(\dfrac{x^2 y^{-3}}{z}\right)^2 = \dfrac{x^4}{y^6 z^2}$

SUGGESTED LEARNING STRATEGIES: Note Taking, Look for a Pattern, Create Representations

TRY THESE D

Simplify and write without negative powers.

a. $(2x^2y)^3 (-3xy^3)^2$

b. $-2ab(5b^2c)^3$

c. $\left(\dfrac{4x}{y^3}\right)^{-2}$

d. $\left(\dfrac{5x}{y}\right)^2 \left(\dfrac{y^3}{10x^2}\right)$

e. $(3xy^{-2})^2(2x^3yz)(6yz^2)^{-1}$

22. The tallest known iceberg in the North Atlantic was measured to be 168 m above sea level, making it the height of a 55-story building. It had an estimated volume of 8.01×10^5 m³, and had an estimated mass of 7.37×10^8 kg. Change these two numbers from **scientific notation** to standard form.

The properties of exponents can be used to multiply and divide numbers expressed in scientific notation.

EXAMPLE 6

Simplify $(1.3 \times 10^5)(4 \times 10^{-8})$.

Step 1: Group terms and use the Product of Powers Property.
$$(1.3 \times 10^5)(4 \times 10^{-8})$$
$$= 1.3 \times 4 \times 10^{5+(-8)}$$

Step 2: Multiply numbers and simplify the exponent.
$$= 5.2 \times 10^{-3}$$

Solution: $(1.3 \times 10^5)(4 \times 10^{-8}) = 5.2 \times 10^{-3}$

My Notes

WRITING MATH

Scientific notation is used to express very large or very small numbers using powers of 10.

$24,000,000 = 2.4 \times 10^7$

$0.0000567 = 5.67 \times 10^{-5}$

Numbers written in scientific notation are always expressed as a product of a number between 1 and 10 and a power of 10.

SUGGESTED LEARNING STRATEGIES: Note Taking

My Notes

EXAMPLE 7

Write in $\dfrac{2.4 \times 10^5}{6 \times 10^7}$ scientific notation.

Step 1:   Group terms and use the $\dfrac{2.4 \times 10^5}{6 \times 10^7} = \dfrac{2.4}{6} \times 10^{5-7}$
        Quotient of Powers Property.

Step 2:   Divide the numbers and
        simplify the exponent. $= 0.4 \times 10^{-2}$

Step 3:   Write 0.4 *in scientific notation* $= 4.0 \times 10^{-1} \times 10^{-2}$

Step 4:   Product of Powers Property. $= 4.0 \times 10^{-1+-2}$

Solution: $\dfrac{2.4 \times 10^5}{6 \times 10^7} = 4.0 \times 10^{-3}$

TRY THESE E

Express the product or quotient using scientific notation.

a. $(2.5 \times 10^{-3})(1.5 \times 10^6)$ **b.** $\dfrac{6.4 \times 10^{23}}{1.6 \times 10^{10}}$

c. Compute the density of the iceberg described in Item 22.

CHECK YOUR UNDERSTANDING

**Write your answers on notebook paper.
Show your work.**

Simplify and write each expression without negative exponents.

1. $x^8 \cdot x^7$

2. $\dfrac{6a^{10}b^9}{3ab^3}$

3. $(6a^2b)(-3ab^3)$

4. $\dfrac{7x^2y^5}{14xy^4}$

5. $(-2z)^{-3}$

6. $\dfrac{6^{-4}}{6^{-2}}$

7. $\dfrac{4x^{-2}}{x^3}$

8. $(5x^5y^{-8}z^2)^0$

9. $(4x^3y^{-1})^2$

10. $\left(\dfrac{5x}{y^2}\right)^3$

11. $(-2a^2b^{-2}c)^3(3ab^4c^5)(xyz)^0$

12. $\dfrac{2xy^2}{x^5y^3} \cdot \dfrac{5xy^3}{-30y^{-2}}$

13. $(2.5 \times 10^3)(5 \times 10^{-3})$ **14.** $\dfrac{6.31 \times 10^7}{2 \times 10^2}$

15. **MATHEMATICAL REFLECTION** What have you learned about simplifying expressions with exponents as a result of this activity?

Exponential Functions
Protecting Your Investment

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations

The National Association of Realtors estimates that, on average, the price of a house doubles every ten years. Tony's grandparents bought a house in 1960 for $10,000. Assume that the trend identified by the National Association of Realtors applies to Tony's grandparents' house.

1. What was the value of Tony's grandparents' house in 1970 and in 1980?

2. Compute the difference in value from 1960 to 1970.

3. Compute the ratio of the 1970 value to the 1960 value.

MATH TiP

The **ratio** of the quantity a to the quantity b is evaluated by dividing a by b $\left(\text{ratio of } a \text{ to } b = \dfrac{a}{b}\right)$.

4. Complete the table of values for the years 1960 to 2010.

House Value				
Year	Decades since 1960	Value of house	Difference between values of consecutive decades	Ratio of values of consecutive decades
1960	0	$10,000	—	—
1970				
1980				
1990				
2000				
2010				

My Notes

My Notes

5. What patterns do you recognize in the table in Item 4?

6. Using the data from the table, graph the ordered pairs (decades since 1960, house value) on the coordinate grid below.

House Value
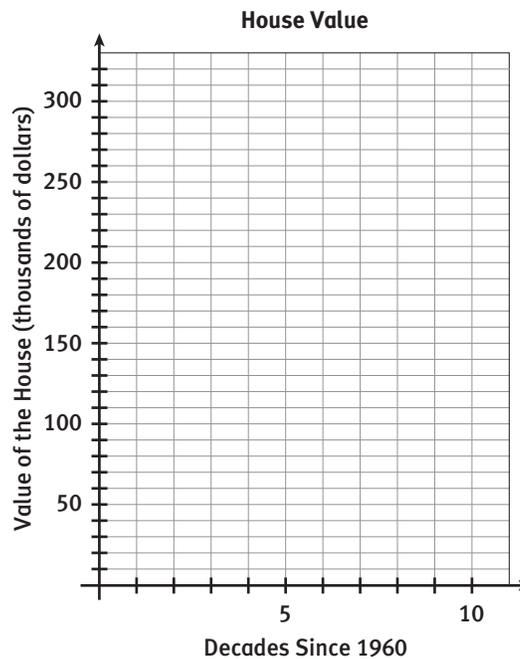
7. Is the data comparing the number of decades since 1960 and value of the house linear? Explain using the table and explain using the graph.

My Notes

8. Using the information that you have regarding the house value, predict the value of the house in the year 2020. Explain how you made your prediction.

9. Tony would like to know what the value of the house was in 2005. Using the same data, predict the house value in 2005. Explain how you made your prediction.

The increase in house value for Tony's grandparents' house is an example of **exponential growth**. Exponential growth can be modeled using an **exponential function**.

Exponential Function

A function of the form $f(x) = a \cdot b^x$,

where x is the domain, $f(x)$ is the range, $a \neq 0$, $b > 0$, and $b \neq 1$.

MATH TERMS
Exponential growth is an increase in a quantity due to multiplying by the same factor during each time period. In a growth, the constant factor is greater than 1.

A function that can be used to model the house value is $h(t) = 10{,}000 \cdot (2)^t$. Use this function for Items 10–12.

10. Identify the meaning of $h(t)$ and t. Which is the domain? Which is the range?

My Notes

TECHNOLOGY TIP

Graph $h(t)$ on a graphing calculator and use trace to find the y coordinate when x is about 2005. The value should be close to your calculated value in Item 12.

CONNECT TO SCIENCE

All radioactive elements have a *half-life*. A half-life is the amount of time in which a radioactive element decays to half of its original quantity.

CONNECT TO SCIENCE

The US Environmental Protection Agency (EPA) recommends that the level of radon be below 4 pCi/L (picoCuries per liter) in any home. The EPA recommends that all homes be tested for radon.

11. Use the function to find the value of the house in the year 2020. How does the value compare with your prediction in Item 8?

12. Use the function to find the value of the house in the year 2005. How does the value compare with your prediction in Item 9?

Radon, a naturally occurring radioactive gas, was identified as a health hazard in some homes in the mid 1980s. Since radon is colorless and odorless, it is important to be aware of the concentration of the gas. Radon has a *half-life* of approximately four days.

Tony's grandparents' house was discovered to have a radon concentration of 400 pCi/L. Renee, a chemist, isolated and eliminated the source of the gas. She then wanted to know the quantity of radon in the house in the days following so that she could determine when the house would be safe.

13. What is the amount of the radon in the house four days after the source was eliminated? Explain your reasoning.

14. Compute the difference of the amount of radon from Day 0 to Day 4.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Self/Peer Revision, Group Presentation

15. Find the ratio of the amount of radon from Day 4 to Day 0.

16. Complete the table for the radon concentration.

Radon Concentration				
Half-Lives	Days after radon source was eliminated	Concentration of radon in pCi/L	Difference between concentration of consecutive half-lives	Ratio of concentrations of consecutive half-lives
0	0	400	—	—
1	4			
2				
3				
4				

17. What patterns do you recognize in the table in Item 16?

My Notes

18. Graph the data in Item 16 as ordered pairs in the form (half lives, concentration).

Radon Concentration

Concentration of Radon (pCi/L)

Half-lives Since Radon Source Was Eliminated
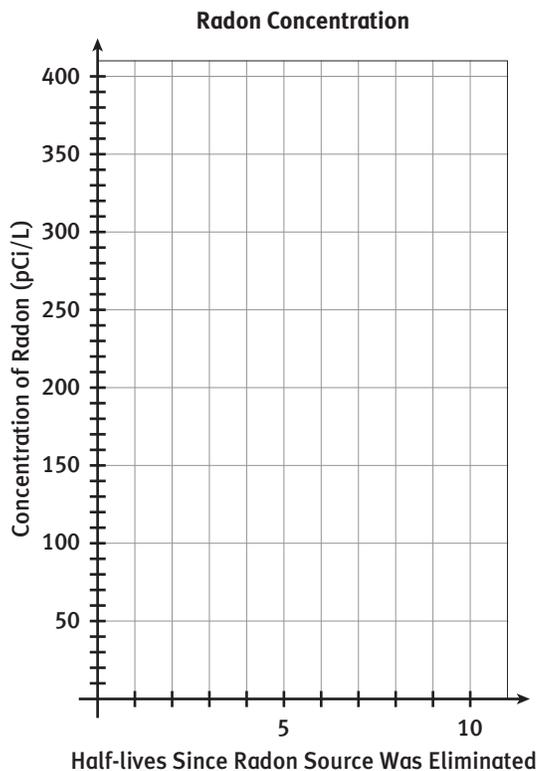
19. Is the data that compares the number of half-lives and the concentration of radon linear? Explain using the table of values and the graph.

20. Renee needs to know the concentration of radon in the house after 20 days. How many radon half-lives are in 20 days? What is the concentration after 20 days?

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Predict and Confirm, Summarize/Paraphrase/Retell, Group Discussion, Visualization

21. How many radon half-lives are in 22 days? Predict the concentration after 22 days.

The decrease in radon concentration in Tony's grandparents' house is an example of **exponential decay**. Exponential decay can also be modeled using an exponential function.

A function that can be used to model the radon concentration is $r(t) = 400 \cdot \left(\frac{1}{2}\right)^t$. Use the function to answer Items 22–24.

22. Identify the meaning of $r(t)$ and t. Which is the domain? Which is the range?

> **MATH TERMS**
>
> **Exponential decay** is a decrease in a quantity due to multiplying by the same factor during each time period. In a decay function, the constant factor is greater than 0 but less than 1.

23. Use the function to find the concentration of radon after 20 days. How does the concentration compare with your prediction in Item 20?

My Notes

24. Use the function to find the concentration of radon after 22 days. How does the concentration compare with your prediction in Items 21?

CONNECT TO AP

In calculus, you will discover what happens as functions approach 0.

25. For the following question, choose always, sometimes, or never. Will the concentration of radon ever be 0? Why or why not?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Make a table of values and graph each function.

   a. $h(x) = 2^x$ **b.** $l(x) = 3^x$

   c. $m(x) = \left(\frac{1}{2}\right)^x$ **d.** $p(x) = \left(\frac{1}{3}\right)^x$

2. Which of the functions in Item 1 represent exponential growth? Explain using your table of values and graph.

3. Which of the functions in Item 1 represent exponential decay? Explain using your table of values and graph.

4. How can you identify which of the functions represent growth or decay by looking at the function?

5. Mold can represent a health hazard in homes. Imagine you are investigating the growth of mold in your science class and are cultivating mold spores in a sample. The table below represents your experimental findings

Number of days	Number of mold spores
0	20
1	80
2	320
3	1280

Write an exponential function $m(t)$ that models the growth of the mold spores, then use the function to predict the number of mold spores on the fifth day and on the eighth day.

6. **MATHEMATICAL REFLECTION** Why can't an exponential function be equal to zero? Why can't an exponential function have a base of one?

Operations with Radicals
Go Fly a Kite

SUGGESTED LEARNING STRATEGIES: Marking Text, Visualization, Debriefing, Creating Representations

Before flying the first airplane in 1903 the Wright Brothers used kites and gliders to study the concepts of aerodynamic forces. The surfaces of a kite generate the forces necessary for flight, and its rigid structures support the surfaces. The frame of a box kite has four "legs" of equal length and four pairs of crossbars, all of equal length, used for bracing the kite. The legs of the kite form a square base around which fabric is wrapped. The crossbars are attached to the legs so that each cross bar is positioned as a diagonal of the square base.

1. a. Label the legs of the kite pictured to the right. How many legs are in a kite? How many cross bars?

b. Label the points on the top view where the ends of the cross bars are attached to the legs A, B, C, and D. Begin at the bottom left and go clockwise.

c. Use one color to show the sides of the square and another color to show crossbar AC. What two figures are formed by two sides of the square and one diagonal?

Members of the Windy Hill Science Club are building kites to explore aerodynamic forces. Club members will provide paper, plastic or light weight cloth for the covering of their kite. The club will provide the balsa wood for the frames.

2. The science club advisor has created the chart below to help determine how much balsa wood he needs to buy.

a. For each kite, calculate the exact length of one crossbar that will be needed to stabilize the kite. Use your drawing from Question 1c as a guide for the rectangular base of these box kites.

Kite	Dimensions of base (in feet)	Exact length of one crossbar (in feet)	Kite	Dimensions of base (in feet)	Exact length of one crossbar (in feet)
A	1 by 1		D	1 by 2	
B	2 by 2		E	2 by 4	
C	3 by 3		F	3 by 6	

b. How much wood would you recommend buying for the cross bars of Kite A? Explain your reasoning.

My Notes

Top View
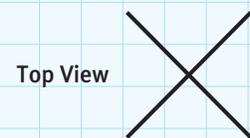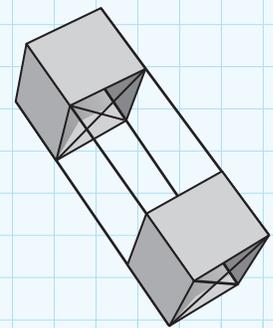
MATH TIP

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

MATH TIP

If you take the square root of a number that is not a perfect square, the result is a decimal number that does not terminate or repeat and is called an **irrational number**.

The exact value of an irrational number must be written using a radical sign.

My Notes

ACADEMIC VOCABULARY

radical expression

MATH TIP

When there is no root index given, it is assumed to be 2 and is called a *square root*.

$$\sqrt{36} = \sqrt[2]{36}$$

READING MATH

$a\sqrt{b}$ is read "*a* times the square root of *b*." Example 1 c. is read "7 times the square root of 12."

To determine how much wood to buy, the club sponsor adds the amounts of wood needed for the kites. Each amount is written as a **radical expression**. Simplifying the expressions will make it easier to add.

> **Radical Expression**
> an expression of the form $\sqrt[n]{a}$, where *a* is the radicand, $\sqrt{}$ is the radical symbol and *n* is the root index.
> $\sqrt[n]{a} = b$, if $b^n = a$ *b* is the *n*th root of *a*.

Finding the square root of a number or expression is the inverse operation of squaring a number or expression.

$$\sqrt{25} = 5, \text{ because } (5)(5) = 25$$
$$\sqrt{81} = 9, \text{ because } (9)(9) = 81$$
$$\sqrt{x^2} = x, \text{ because } (x)(x) = x^2, x \geq 0$$

Notice also that $(-5)(-5) = (-5)^2 = 25$. The **principal square root** of a number is the positive square root value. The expression $\sqrt{25}$ simplifies to 5, the principal square root. The **negative square root** is the negative root value, so $-\sqrt{25}$ simplifies to -5.

To simplify square roots in which the radicand is not a perfect square:

Step 1: Write the radicand as a product of numbers, one of which is a perfect square.

Step 2: Find the square root of the perfect square.

EXAMPLE 1

Simplify each expression.

a. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

b. $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$ or
$\sqrt{72} = \sqrt{9 \cdot 4 \cdot 2} = (3 \cdot 2)\sqrt{2} = 6\sqrt{2}$

c. $7\sqrt{12} = 7\sqrt{4 \cdot 3} = 7(2\sqrt{3}) = 14\sqrt{3}$

d. $\sqrt{c^3} = \sqrt{c^2 \cdot c} = c\sqrt{c}, c \geq 0$

TRY THESE A

Simplify each expression.

a. $\sqrt{18}$ **b.** $5\sqrt{48}$ **c.** $\sqrt{126}$ **d.** $\sqrt{24y^2}$ **e.** $\sqrt{45b^3}$

SUGGESTED LEARNING STRATEGIES: Create Representations

3. Copy the lengths of the crossbars from the chart in Item 1. Then express the lengths of the crossbars in simplified form.

Kite	Dimensions of base (feet)	Exact length of one cross bar (feet)	Simplified form of length of crossbar
A	1 by 1		
B	2 by 2		
C	3 by 3		
D	1 by 2		
E	2 by 4		
F	3 by 6		

The club advisor wants to find the total length of the balsa wood needed. To do so, he will need to add radicals.

Addition Property of Radicals
$$a\sqrt{b} \pm c\sqrt{b} = (a \pm c)\sqrt{b},$$
where $b \geq 0$.

To add or subtract radicals, the index and radicand must be the same.

EXAMPLE 2

Add or subtract each expression and simplify.

a. $3\sqrt{5} + 7\sqrt{5}$

$= (3 + 7)\sqrt{5}$ ← Add or subtract →

$= 10\sqrt{5}$ the coefficients.

b. $10\sqrt{3} - 4\sqrt{3}$

$= (10 - 4)\sqrt{3}$

$= 6\sqrt{3}$

c. $2\sqrt{5} + 8\sqrt{3} + 6\sqrt{5} - 3\sqrt{3}$

Step 1: *Group terms with like radicands* $= 2\sqrt{5} + 6\sqrt{5} + 8\sqrt{3} - 3\sqrt{3}$

Step 2: *Add or subtract the coefficients.* $= (2 + 6)\sqrt{5} + (8 - 3)\sqrt{3}$

$= 8\sqrt{5} + 5\sqrt{3}$

Solution: $2\sqrt{5} + 8\sqrt{3} + 6\sqrt{5} - 3\sqrt{3} = 8\sqrt{5} + 5\sqrt{3}$

My Notes

My Notes

TRY THESE **B**

Add or subtract each expression and simplify.

a. $2\sqrt{7} + 3\sqrt{7}$ **b.** $5\sqrt{6} + 2\sqrt{5} - \sqrt{6} + 7\sqrt{5}$

c. $2\sqrt{2} + \sqrt{8} + 3\sqrt{2}$

4. The club advisor also needs to know how much wood to buy for the legs of the kites. Each kite will be 3 ft tall.

a. Complete the table below.

Kite	Dimensions of base (feet)	Length of one crossbar (feet)	Length of one leg (feet)	Wood needed for legs (feet)	Wood needed for crossbars (feet)
A	1 by 1				
B	2 by 2				
C	3 by 3				
D	1 by 2				
E	2 by 4				
F	3 by 6				

b. How much balsa wood should the club advisor buy if the club is going to build the six kites described above?

c. Explain how you reached your conclusion.

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Quickwrite, Predict and Confirm

5. a. Complete the table below and simplify the radical expressions Column 3 and Column 5.

a	b	$\sqrt{a} \cdot \sqrt{b}$	ab	$\sqrt{ab}$
4	9			
100	25			
9	16			

b. Use the patterns you observe in the table above to write an equation that relates $\sqrt{a}, \sqrt{b}$, and $\sqrt{ab}$.

c. All the values of a and b in Item 5(a) are perfect squares. Choose some values for a and b that are *not* perfect squares and use a calculator to show that the equation you wrote in Item 5(b) is true for those numbers as well.

a	b	$\sqrt{a} \cdot \sqrt{b}$	ab	$\sqrt{ab}$

TECHNOLOGY TIP

Approximate values of square roots that are not perfect squares can be found using a calculator.

d. Simplify the products in Columns A and B.

A	Simplified form	B	Simplified form
$(2\sqrt{4})(\sqrt{9})$		$2\sqrt{4 \cdot 9}$	
$(3\sqrt{4})(5\sqrt{16})$		$(3 \cdot 5)\sqrt{4 \cdot 16}$	
$(2\sqrt{7})(3\sqrt{14})$		$(2 \cdot 3)(\sqrt{7 \cdot 14})$	

My Notes

e. Write a verbal rule that explains how to multiply radical expressions.

Multiplication Property of Radicals

$$(a\sqrt{b})(c\sqrt{d}) = ac\sqrt{bd},$$

where $b \geq 0$, $d \geq 0$.

To multiply radical expressions, the index must be the same. Find the product of the coefficients and the product of the radicands. Simplify the radical expression.

EXAMPLE 3

Multiply each expression and simplify.

a. $(3\sqrt{6})(4\sqrt{5}) = (3 \cdot 4)(\sqrt{6 \cdot 5}) = 12\sqrt{30}$

b. $(2\sqrt{10})(3\sqrt{6})$
$= (2 \cdot 3)\sqrt{10 \cdot 6}$ *Step 1:* Multiply
$= 6\sqrt{60}$
$= 6(\sqrt{4 \cdot 15})$ *Step 2:* Simplify
$= (6 \cdot 2)\sqrt{15}$
$= 12\sqrt{15}$

c. $(2x\sqrt{6x})(5\sqrt{3x^2})$
$= 10x\sqrt{6x \cdot 3x^2}$
$= 10x(\sqrt{18x^3})$
$= 10x(\sqrt{9x^2 \cdot 2x})$
$= (10x)(3x)(\sqrt{2x})$
$= 30x^2\sqrt{2x}$

TRY THESE **C**

Multiply each expression and simplify.

a. $(2\sqrt{10})(5\sqrt{3})$
b. $(3\sqrt{8})(2\sqrt{6})$
c. $(4\sqrt{12})(5\sqrt{18})$
d. $(3\sqrt{5a})(2a\sqrt{15a^2})$

Division Property of Radicals

$$\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}},$$

where $b \geq 0$, $d > 0$.

CONNECT TO AP

In later mathematics courses, you will study another system of numbers, called the complex numbers. In the complex number system, $\sqrt{-1}$ is defined.

My Notes

To divide radical expressions, the index must be the same. Find the quotient of the coefficients and the quotient of the radicands. Simplify the expression.

EXAMPLE 4

Divide each expression and simplify.

a. $\dfrac{\sqrt{6}}{\sqrt{2}} = \sqrt{\dfrac{6}{2}} = \sqrt{3}$

b. $\dfrac{2\sqrt{10}}{3\sqrt{2}} = \dfrac{2}{3}\sqrt{\dfrac{10}{2}} = \dfrac{2}{3}\sqrt{5}$

c. $\dfrac{8\sqrt{24}}{2\sqrt{3}} = \dfrac{8}{2}\sqrt{\dfrac{24}{3}} = 4\sqrt{8} = 4\sqrt{4 \cdot 2} = 4(2\sqrt{2}) = 8\sqrt{2}$

TRY THESE **D**

Divide each expression and simplify.

a. $\dfrac{\sqrt{22}}{\sqrt{2}}$ **b.** $\dfrac{4\sqrt{42}}{5\sqrt{6}}$ **c.** $\dfrac{10\sqrt{54}}{2\sqrt{2}}$ **d.** $\dfrac{12\sqrt{75}}{3\sqrt{3}}$

MATH TERMS
Rationalize means to make rational. You can **rationalize the denominator** without altering the value of the expression by multiplying the fraction by an appropriate form of 1.

A radical expression in simplified form does not have a radical in the denominator. Most frequently, the denominator is **rationalized**. You **rationalize the denominator** by simplifying the expression to get a perfect square under the radicand in the denominator.

$$\frac{\sqrt{a}}{\sqrt{b}} \cdot 1 = \left(\frac{\sqrt{a}}{\sqrt{b}}\right)\left(\frac{\sqrt{b}}{\sqrt{b}}\right) = \frac{\sqrt{ab}}{\sqrt{b^2}} = \frac{\sqrt{ab}}{b}$$

CONNECT TO AP

In calculus, both numerators and denominators are rationalized. The procedure for rationalizing a numerator is similar to that for rationalizing a denominator.

EXAMPLE 5

Rationalize the denominator. $\dfrac{\sqrt{5}}{\sqrt{3}}$

Step 1: *Multiply the numerator and denominator by* $\sqrt{3}$. $\dfrac{\sqrt{5}}{\sqrt{3}} = \dfrac{\sqrt{5}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{15}}{\sqrt{9}}$

Step 2: *Simplify.* $= \dfrac{\sqrt{15}}{3}$

Solution: $\dfrac{\sqrt{5}}{\sqrt{3}} = \dfrac{\sqrt{15}}{3}$

My Notes

TRY THESE **E**

Rationalize the denominator in each expression.

a. $\dfrac{\sqrt{11}}{\sqrt{6}}$ **b.** $\dfrac{2\sqrt{7}}{\sqrt{5}}$ **c.** $\dfrac{3\sqrt{5}}{\sqrt{8}}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Express each expression in simplest radical form.

1. $\sqrt{40}$ **2.** $\sqrt{128}$

3. $\sqrt{162}$ **4.** $4\sqrt{27} + 6\sqrt{8}$

5. $8\sqrt{6} + 2\sqrt{12} + 5\sqrt{3} - \sqrt{54}$

6. $8\sqrt{98} + 3\sqrt{32} - 2\sqrt{75}$

7. $(4\sqrt{7})(2\sqrt{3})$ **8.** $\sqrt{2}\,(\sqrt{2} + 3\sqrt{6})$

9. $\dfrac{\sqrt{75}}{\sqrt{5}}$ **10.** $\sqrt{\dfrac{5}{8}}$

11. $\left(\sqrt{\dfrac{1}{2}}\right)\left(\sqrt{\dfrac{3}{5}}\right)$ **12.** $\sqrt{27} + \sqrt{\dfrac{1}{27}}$

13. Find the difference of $9\sqrt{20}$ and $2\sqrt{5}$.

A. $7\sqrt{15}$ **B.** $16\sqrt{5}$

C. $9\sqrt{5}$ **D.** $7\sqrt{5}$

14. Find the product of $(2 + \sqrt{3})$ and $(\sqrt{6} + \sqrt{8})$.

A. $4\sqrt{6}$

B. $4 + 2\sqrt{3} + 3\sqrt{2} + 2\sqrt{6}$

C. $6\sqrt{6} + 7\sqrt{2}$

D. $7\sqrt{2} + 4\sqrt{6}$

15. The time, T, in seconds, it takes the pendulum of a clock to swing from one side to the other side is given by the formula $T = \pi\sqrt{\dfrac{l}{32}}$ where l is the length of the pendulum, in feet. The clock ticks each time the pendulum is at the extreme left or right point.

a. If the pendulum is 4 feet long, how long does it take the pendulum to swing from left to right? Give an exact value in terms of π.

b. If the pendulum is shortened will the clock tick more or less often? Explain how you arrived at your conclusion.

16. **MATHEMATICAL REFLECTION** What conditions must be satisfied for a radical expression to be in simplified form?

Exponential Functions and Radicals

DECISIONS, DECISIONS

Carlos is looking to spend up to $7000 on his first car. He's narrowed his choices to two different vehicles. The first vehicle is a three-year old sports car with a sale price of $7000. The second vehicle is a classic 1956 Chevy Bel-Air his neighbor is selling for $3125. Whichever car he buys, he plans to keep until he graduates from college in seven years.

1. The value of the sports car will depreciate by 12% each year.

   a. Write a function that will allow Carlos to determine the value of the sports car after each year.

   b. Use your function to determine the value of the sports car seven years from now. Round your answer to the nearest dollar.

2. If kept in good condition, the value of the Bel-Air will appreciate by 8% each year.

   a. Write a function that will allow Carlos to determine the value of the car after each year.

   b. Use your function to determine the value of the Bel-Air seven years from now. Round your answer to the nearest dollar.

3. Sketch a graph of the value of each car over the next seven years.

4. During which year will the values of each car be the same? Explain two different methods you could use to determine your answer.

If Carlos chooses to buy the Bel-Air, he plans to rent a storage unit during December and January of each year in order to preserve the condition of the car. Store More, Inc. offers a 14' × 16' unit, and X-tra Space Enterprises offers a 12' × 18' unit.

5. Each company bases its monthly rental price on the hypotenuse of the floor of the unit.

   a. Determine the hypotenuse of the floor of the Store More, Inc. unit in simplest radical form.

   b. Determine the hypotenuse of the floor of the X-tra Space Enterprises unit in simplest radical form.

Exponential Functions and Radicals

DECISIONS, DECISIONS

6. The sale price of the Bel-Air, $3125, can also be written as 5^5. Use the laws of exponents listed below to provide an example of a problem that would produce an answer of 5^5. Justify your reasoning for each example.

a. $a^m \cdot a^n$ **b.** $\dfrac{a^m}{a^n}$ **c.** $\dfrac{1}{a^n}$ **d.** $(a^m)^n$

	Exemplary	Proficient	Emerging
Math Knowledge #1b, 2b, 6a, b, c, d	The student: • Determines the correct value of both cars and rounds the answer correctly. (1b, 2b) • Writes correct examples for the expressions given. (6a, b, c, d) • Gives the correct rules of exponents. (6a, b, c, d)	The student: • Determines the correct value of one of the cars and rounds the answer correctly; attempts to determine the value of the other car. • Writes correct examples for only three of the expressions given. • Gives correct rules of exponents for at least two items.	The student: • Attempts to determine the correct value of the cars, but neither answer is correct. • Writes a correct example for only two of the expressions given. • Gives correct rules of exponents for only one item.
Problem Solving #1a, 2a, 4, 5a, b	The student: • Writes a correct function for the yearly value of both cars. (1a, 2a) • Correctly determines the year the values will be the same. (4) • Correctly determines the hypotenuse of both floors. (5a, b)	The student: • Writes a correct function for the yearly value of one of the cars. • Correctly determines the year the values will be the same, based on the graphs drawn. • Uses the correct method to determine the hypotenuse but makes computational errors.	The student: • Attempts to write the functions, but neither is correct. • Gives an incorrect year. • Attempts to determine the hypotenuse, but the method used is incorrect.
Representations #3	The student correctly sketches the graphs of both of the given functions. (3)	The student correctly sketches the graph of only one of the given functions.	The student attempts to sketch the graphs of the functions, but neither is correct.
Communication #4	The student explains two methods that can be used to determine the correct year the values of the cars would be the same. (4)	The student explains one method that can be used to determine the correct year the values of the cars would be the same; the second explanation is incomplete, but contains no mathematical errors.	The student attempts to explain the methods, but both explanations are incomplete or contain errors.

Adding and Subtracting Polynomials
## Polynomials in the Sun

SUGGESTED LEARNING STRATEGIES: Shared Reading, Questioning the Text, Create Representations, Vocabulary Organizer, Note Taking

A solar panel is a device that collects and converts solar energy into electricity or heat. The solar panel consists of interconnected solar cells. The panels can have differing numbers of solar cells and can come in square or rectangular shapes.

1. How many solar cells are in the panel below?
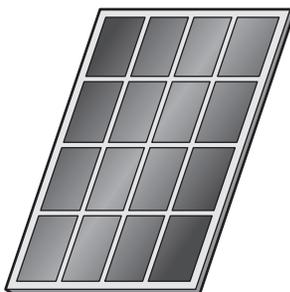
2. If a solar panel has four rows as the picture does, but can be extended to have an unknown number of columns, x, write an expression to give the number of solar cells that could be in the panel.

3. If a solar panel could have x rows and x columns of solar cells, write an expression that would give the total number of cells in the panel.

4. If you had 5 panels like those found in Item 3, write an expression that would give the total number of solar cells.

All the answers in Items 1–4 are called *terms*. A **term** is a number, variable or the product of a number and variable(s).

5. Write the sum of your answers from Items 1, 2, and 4.

> **CONNECT TO SCIENCE**
>
> Solar panels, also known as photovoltaic panels, are made of semiconductor materials. A panel has both positive and negative layers of semiconductor material. When sunlight hits the semiconductor, electrons travel across the intersection of the two different layers of materials, creating an electric current.

> **ACADEMIC VOCABULARY**
>
> term

My Notes

ACADEMIC VOCABULARY

A **polynomial** is a term or the sum of two or more terms.

ACADEMIC VOCABULARY

A **coefficient** is the numeric factor of a term.

MATH TERMS

A **constant term** is a term that contains only a number, such as your answer to Item 1. The constant term of a polynomial is a term of degree zero. For example, 4 can also be written as $4x^0$

Expressions like the one in Item 5 are called **polynomials**. A **polynomial** is a single term or the sum of two or more terms.

6. List the terms of the polynomial you found in Item 5.

7. What are the **coefficients** and **constant** terms of the polynomial in Item 5?

The **degree of a term** is the sum of the exponents on the variables contained in the term.

8. Find the degree and coefficient of each term in the polynomial $4x^5 + 12x^3 + x^2 - x + 5$.

Term	Degree	Coefficient
$4x^5$	5	
$12x^3$		12
x^2		
$-x$		
5		

9. For the polynomial $2x^3y - 6x^2y^2 + 9xy - 13y^5 + 5x + 15$, list each term and identify its degree and coefficient.

# Adding and Subtracting Polynomials
Polynomials in the Sun

SUGGESTED LEARNING STRATEGIES: Note Taking, Vocabulary Organizer, Interactive Word Wall

The **degree of a polynomial** is the largest degree of any term in the polynomial.

10. Find the degree and constant term of each polynomial.

Polynomial	Degree of Polynomial	Constant Term
$2x^2 + 3x + 7$	2	
$-5y^3 + 4y^2 - 8y - 3$		
$36 + 12x + x^2$		36

The **standard form of a polynomial** is a polynomial written in **descending order** of degree. The **leading coefficient** is the coefficient of a polynomial's leading term when it is written in standard form.

Degree of the polynomial

Leading coefficient $\longrightarrow$ $7x^5 + 2x^2 - 3$ $\longleftarrow$ Constant
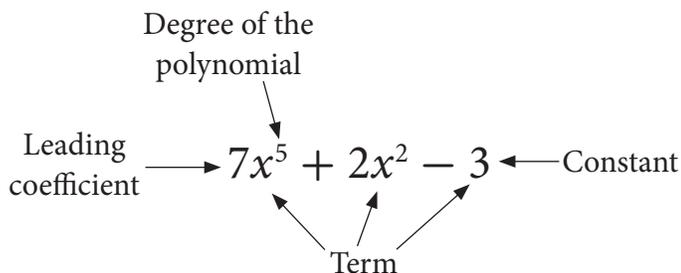
Term

A polynomial can be classified by the number of terms it has when it is in simplest form.

Name	Number of Terms n	Examples
monomial	1	8 or $-2x$ or $3x^2$
binomial	2	$3x + 2$ or $4x^2 - 7x$
trinomial	3	$-x^2 - 3x + 9$
polynomial	$n > 3$	$9x^4 - x^3 - 3x^2 + 7x - 2$

My Notes

ACADEMIC VOCABULARY

degree of a polynomial

MATH TERMS
Descending order of degree means that the term that has the highest degree is written first, the term with the next highest degree is written next, and so on.

READING MATH
The prefixes mono (one), bi (two), tri (three) and poly (many) appear in many math terms such as bisect (cut in half), triangle (three-sided figure), polygon (many-sided) figure.

**SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revision**

My Notes

11. Fill in the missing information in the table below.

Polynomial	Number of Terms	Name	Leading coefficient	Constant term	Degree
$3x^2 - 5x$					
$-2x^2 + 13x + 6$					
$15x^2$					
$5p^3 + 2p^2 - p - 7$					
$a^2 - 25$					
$0.23x^3 + 0.54x^2 - 0.58x + 0.0218$					
$-9.8t^2 - 20t + 150$					

12. If a square solar panel with an unknown number of cells along the edge can be represented by x^2, how many cells would be in one column of the panel?

A square solar panel with x rows and x columns can be represented by the algebra tile:
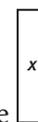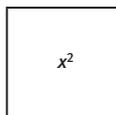
A column of x cells can be represented by using the tile , and a single solar cell can be represented by .

Suppose there were 3 square solar panels that each had x columns and x rows, 2 columns with x cells, and 3 single solar cells. You can represent $3x^2 + 2x + 3$ using algebra tiles.

SUGGESTED LEARNING STRATEGIES: Group Discussion

13. Represent $2x^2 - 3x + 2$ using algebra tiles. Draw a picture of the representation below.

My Notes

Adding Polynomials

Adding polynomials using algebra tiles can be done by:

- modeling each polynomial
- identifying and removing zero pairs
- writing the new polynomial

---

EXAMPLE 1

Add $(3x^2 - 3x - 5) + (2x^2 + 5x + 3)$ using algebra tiles.

Step 1: Model the polynomials.
$3x^2 - 3x - 5 \qquad 2x^2 + 5x + 3$
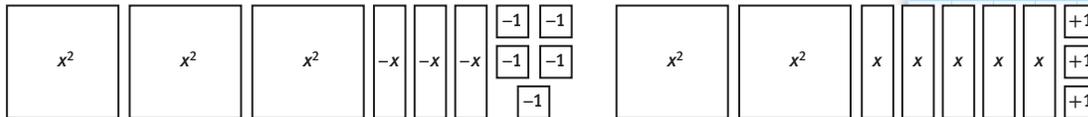
Step 2: Identify and remove zero pairs.
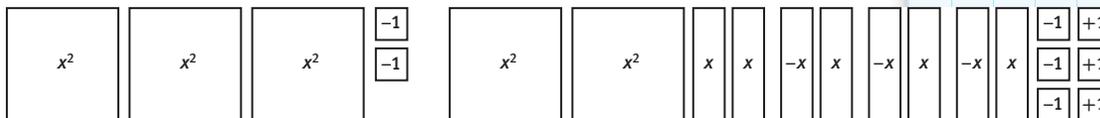
Step 3: Combine like tiles.
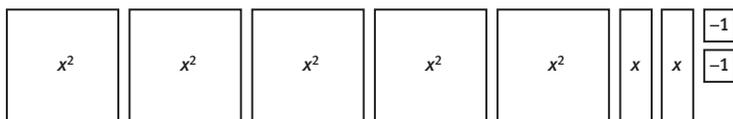
Step 4: Write the polynomial for the model in Step 3.
$5x^2 + 2x - 2$

Solution: $(3x^2 - 3x - 5) + (2x^2 + 5x + 3) = 5x^2 + 2x - 2$

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Use Manipulatives

TRY THESE **A**

Add using algebra tiles.

a. $(x^2 - 2) + (2x^2 + 5)$

b. $(2y^2 + 3y + 6) + (3y^2 - 4)$

c. $(2x^2 + 3x + 9) + (-x^2 - 4x - 6)$

d. $(5 - 3x + x^2) + (2x + 4 - 3x^2)$

14. Can you use algebra tiles to add $(4x^4 + 3x^2 + 15) + (x^4 + 10x^3 - 4x^2 + 22x - 23)$? If so, model the polynomials and add. If not, explain why.

Like terms in an expression are the terms that have the same variable and exponent for that variable. All constants are like terms.

15. State whether the terms are like or unlike terms. Explain.

   a. $2x$; $2x^3$

   b. 5; $5x$

   c. $-3y$; $3y$

   d. x^2y; xy^2

   e. 14; -0.6

16. Using vocabulary from this unit, describe a method that could be used to add polynomials without using algebra tiles.

SUGGESTED LEARNING STRATEGIES: Note Taking

The method you described in Item 16 can be used to add polynomials algebraically.

EXAMPLE 2

Add $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3)$ horizontally and vertically.

<u>Horizontally</u>

Step 1: Find like terms. $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3)$

Step 2: Group like terms. $= (3x^3) + (2x^2 + 4x^2) + (-5x + 2x) + (7 - 3)$

Step 3: Add the coefficients of like terms. $= 3x^3 + 6x^2 - 3x + 4$

Solution: $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3) = 3x^3 + 6x^2 - 3x + 4$

<u>Vertically</u>

Step 1: Vertically align the like terms.

Step 2: Add the coefficients of like terms.

$$3x^3 + 2x^2 - 5x + 7$$
$$\underline{+\ 4x^2 + 2x - 3}$$
$$3x^3 + 6x^2 - 3x + 4$$

Solution: $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3) = 3x^3 + 6x^2 - 3x + 4$

MATH TIP

Recall that the commutative and associative properties of addition allow you to re-order and group like terms.

TRY THESE B

Add using either the horizontal or the vertical method.

a. $(4x^2 + 3) + (x^2 - 3x + 5)$

b. $(10y^2 + 8y + 6) + (17y^2 - 11)$

c. $(9x^2 + 15x + 21) + (-13x^2 - 11x - 26)$

d. $(18 + 21x^2) + (3x + 4 - 52x^2)$

Adding and Subtracting Polynomials
Polynomials in the Sun

My Notes

Subtracting Polynomials

Subtracting polynomials using algebra tiles can be done by

- modeling the first polynomial
- modeling the additive inverse of the second polynomial
- identifying and removing zero pairs
- writing the new polynomial

MATH TIP

Remember that subtraction can be represented by adding the opposite, the additive inverse.

EXAMPLE 3

Subtract $(2x^2 + x - 3) - (x^2 + 4x + 1)$ using algebra tiles.

Step 1: Model $2x^2 + x - 3$ and the additive inverse of $x^2 + 4x + 1$.
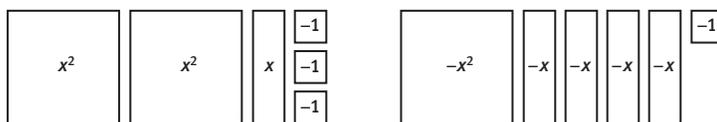
Step 2: Identify and remove zero pairs.
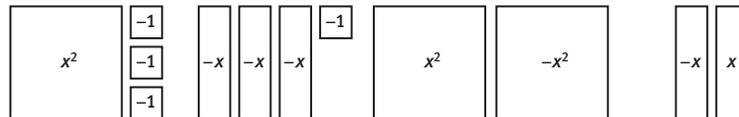
Step 3: Combine like tiles.
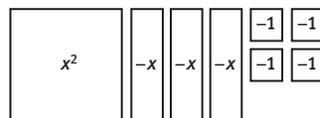
Step 4: Write the polynomial for the model in Step 4. $x^2 - 3x - 4$

Solution: $(2x^2 + x - 3) - (x^2 + 4x + 1) = x^2 - 3x - 4$

TRY THESE C

a. Use algebra tiles to represent the additive inverse of $3x^2 - 2x - 4$. Then draw the representation.

SUGGESTED LEARNING STRATEGIES: Note Taking

TRY THESE C (*continued*)

Subtract using algebra tiles.

b. $(3x^2 - 1) - (2x^2 + 6)$

c. $(3y^2 + 4y + 7) - (2y^2 - 2)$

d. $(x^2 + x + 2) - (-2x^2 - 5x - 7)$

e. $(4 + 2x + 4x^2) - (2x + 4 - 3x^2)$

To subtract a polynomial you add its opposite, or subtract each of its terms.

EXAMPLE 4

Subtract $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6)$ horizontally and vertically.

<u>Horizontally</u>

Step 1:   *Distribute the* $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6)$
         negative.

Step 2:   *Find like terms.* $= 2x^3 + 8x^2 + x + 10 - 5x^2 + 4x - 6$

Step 3:   *Group like terms* $= 2x^3 + (8x^2 - 5x^2) + (x + 4x) + (10 - 6)$

Step 4:   *Combine* $= 2x^3 + 3x^2 + 5x + 4$
         coefficients of
         like terms.

Solution: $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6) = 2x^3 + 3x^2 + 5x + 4$

<u>Vertically</u>

Step 1:   *Vertically align the like terms.* $2x^3 + 8x^2 + x + 10$
                                        $\underline{-(5x^2 - 4x + 6)}$
                                        $2x^3 + 8x^2 + x + 10$

Step 2:   *Distribute the negative.* $\underline{\quad -5x^2 + 4x - 6}$

Step 3:   *Combine coefficients of like terms.* $2x^3 + 3x^2 + 5x + 4$

Solution: $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6) = 2x^3 + 3x^2 + 5x + 4$

My Notes

> **MATH TIP**
>
> To multiply a monomial by a polynomial, use the distributive property. Multiply each term of the trinomial by the monomial.

TRY THESE D

Subtract using either the horizontal or the vertical method.

a. $(5x^2 - 5) - (x^2 + 7)$

b. $(2x^2 + 3x + 2) - (-5x^2 - 2x - 9)$

c. $(y^2 + 3y + 8) - (4y^2 - 9)$

d. $(12 + 5x + 14x^2) - (8x + 15 - 7x^2)$

17. Suppose there are 10 solar panels that have $3x^2 + 7x + 3$ cells on each panel. Write a polynomial that represents the total number of solar cells in all 10 panels combined.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

For Items 1–5, use the polynomial $4x^3 + 3x^2 - 9x + 7$.

1. Name the coefficients of the polynomial

2. List the terms, and specify the degree of each term.

3. What is the degree of the polynomial?

4. What is the leading coefficient of the polynomial?

5. What is the constant term of the polynomial?

Find the sum or difference.

6. $(3x^2 + 2x + 7) + (-2x^3 + 3x^2 - 8x - 6)$

7. $(2x^3 + 4x - 9) - (-7x^3 + 2x^2 - 5x + 13)$

8. MATHEMATICAL REFLECTION Describe the similarities and differences in adding and subtracting polynomials. What method of adding polynomials do you feel most comfortable with?

Multiplying Polynomials
Tri-Com Computers

SUGGESTED LEARNING STRATEGIES: Shared Reading, Visualization, Think/Pair/Share, Look for a Pattern

Tri-Com Consulting is a company that sets up local area networks in offices. In setting up a network, consultants need to consider not only where to place computers, but also where to place peripheral equipment, such as printers.

Tri-Com typically sets up local area networks of computers and printers in square or rectangular offices. Printers are placed in each corner of the room. The primary printer A serves the first 25 computers and the other three printers, B, C, and D, are assigned to other regions in the room. Below is an example.
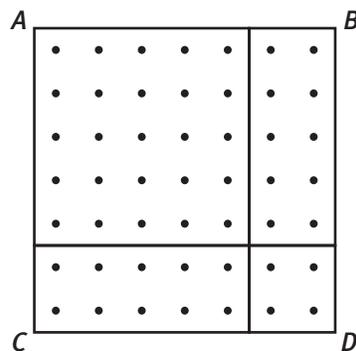

1. If each dot represents a computer, how many computers in this room will be assigned to each of the printers?

2. What is the total number of computers in the room? Describe two ways to find the total.

My Notes

Another example of an office in which Tri-Com installed a network had 9 computers along each wall. The computers are aligned in an array with the number of computers in each region determined by the number of computers along the wall.
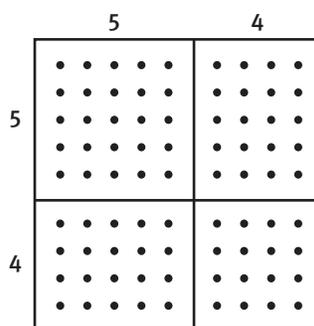
3. A technician claimed that since $9 = 5 + 4$, the number of computers in an office could be written as an expression using only the numbers 5 and 4. Is the technician correct? Explain.

4. Show another way to determine the total number of computers in the office.

5. Use the diagram above and the distributive property to explain why the expression $(5 + 4)(5 + 4)$ could be used to find the total number of computers.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Graphic Organizer, Activating Prior Knowledge

6. The office to the right has 8^2 computers. Fill in the number of computers in each section if it is split into a $(5 + 3)^2$ configuration.
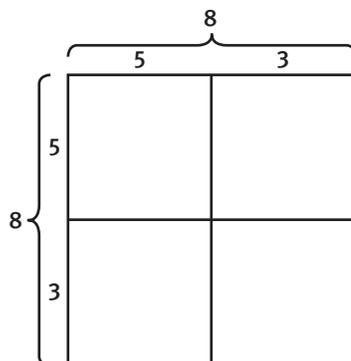
7. What is the total number of computers? Describe two ways to find the total.

8. For each possible office configuration below, draw a diagram like the one next to Item 6. Label the number of computers on the edge of each section and find the total number of computers in the room by adding the number of computers in each section.

a. $(2 + 3)^2$

b. $(4 + 1)^2$

c. $(3 + 7)^2$

My Notes

Tri-Com has a minimum requirement of 25 computers per installation arranged in a 5 by 5 array. Some rooms are larger than others and can accommodate more than 5 computers along each wall to complete a square array. It is helpful to use a variable expression to represent the total number of computers needed for any office having x more than the 5 computer minimum along each wall.
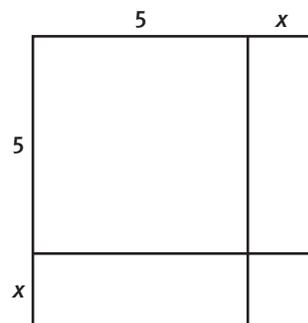
9. One technician said that $5^2 + x^2$ would be the correct way to represent the total number of computers in the office space. Use the diagram to explain how the statement is incorrect.

10. Write an expression for the sum of the number of computers in each region in Item 9.

11. For each of the possible room configurations, find the total number of computers in the room.

   a. $(2 + x)^2$

   b. $(x + 3)^2$

   c. $(x + 6)^2$

SUGGESTED LEARNING STRATEGIES: Create Representations

The graphic organizer below can be used to help arrange the multiplications of the distributive property. It does not need to be related to the number of computers in an office. For example, this graphic organizer shows $5 \cdot 7 = (3 + 2)(4 + 3)$.
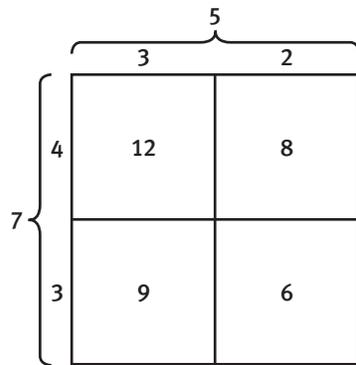
12. Draw a graphic organizer to represent the expression $(5 + 2)(2 + 3)$. Label each inner rectangle and find the sum.

13. Draw a graphic organizer to represent the expression $(6 - 3)(4 - 2)$. Label each inner rectangle and find the sum.

My Notes

14. Multiply the binomials in Item 13 using the distributive property. What do you notice?

You can use the same graphic organizer to multiply binomials that contain variables. The following diagram represents $(x - 2)(x - 3)$.
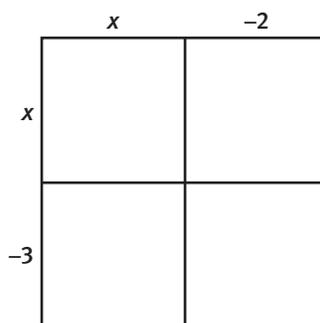
15. Use the graphic organizer above to represent the expression $(x - 2)(x - 3)$. Label each inner rectangle and find the sum.

16. Multiply the binomials in Item 15 using the distributive property. What do you notice?

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Quickwrite, Create Representations

17. Find the product of the binomials. If you want, draw a graphic organizer like the one above Item 15.

a. $(x - 7)(x - 5)$

b. $(8 + x)(4 - x)$

c. $(x + 7)(x + 5)$

d. $(x + y)^2$

18. Compare and contrast the use of the graphic organizer and the use of the distributive property to find the product of two binomials.

19. Find the product of the two binomials.

a. $(x + 5)(x - 5)$

b. $(4 + x)(4 - x)$

c. $(x - 7)(x + 7)$

d. $(2x - 3)(2x + 3)$

Exploring a cave.

My Notes

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite

20. The product of binomials of the form $(a + b)(a - b)$, has a special pattern called a **difference of two squares**. Use this pattern to explain how to find the product of $(a + b)(a - b)$.

21. Find the product of the two binomials. Look for a pattern.

a. $(x + 5)^2$

b. $(4 + x)^2$

c. $(x + 7)^2$

d. $(2x + 3)^2$

e. $(x - 5)^2$

f. $(4 - x)^2$

g. $(x - 7)^2$

h. $(2x - 3)^2$

Multiplying Polynomials
Tri-Com Computers

SUGGESTED LEARNING STRATEGIES: Graphic Organizer,
Create Representations

My Notes

22. The **square of a binomial,** $(a + b)^2$ or $(a - b)^2$, also has a special pattern. Look for a pattern in the products you found for Item 21. Use this pattern to explain how to find the square of any binomial.

A graphic organizer can also be used to multiply polynomials that have more than two terms, such as a binomial times a trinomial. The graphic organizer at the right can be used to multiply $(x + 2)(x^2 + 2x + 3)$.

	x	2
x^2	x^3	$2x^2$
$2x$	$2x^2$	$4x$
3	$3x$	6

$= x^3 + 4x^2 + 7x + 6$

> **MATH TERMS**
> Trinomials of the form
> $a^2 + 2ab + b^2$ and
> $a^2 - 2ab + b^2$ are known as
> **perfect square trinomials.**
> This type of trinomial can be factored in the special pattern
> $(a + b)^2$ for $a^2 + 2ab + b^2$ or
> $(a - b)^2$ for $a^2 - 2ab + b^2$.
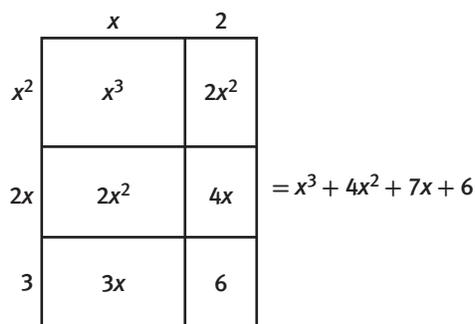
23. Draw a graphic organizer in the My Notes section to represent the expression $(x - 3)(x^2 + 5x + 6)$. Label each inner rectangle and find the sum.

24. How many boxes would you need to represent the multiplication of $(x^3 + 5x^2 + 3x - 3)(x^4 - 6x^3 - 7x^2 + 5x + 6)$ using the graphic organizer?
    a. Explain how you determined your answer.
    b. Would you use the graphic organizer for other multiplications with this many terms? Why/why not?

The distributive property can be used to multiply any size polynomial by another. Multiply each term in the first polynomial by each term in the second polynomial.

$$(x - 3)(5x^2 - 2x + 1) = 5x^3 - 17x^2 + 7x - 3$$

My Notes

25. Find the product of the two polynomials.

    a. $x(x + 5)$

    b. $(x - 3)(x + 6)$

    c. $(x + 7)(3x^2 - x - 1)$

    d. $(3x - 7)(4x^2 + 4x - 3)$

26. How can you predict the number of terms the product should have before you combine like terms?

CHECK YOUR UNDERSTANDING

**Write your answers on notebook paper.
Show your work.**

Find each product.

1. $(8 + 9)^2$

2. $(11 + 6)(9 + 2)$

3. $(x + 12)(x - 10)$

4. $(x + 9)(x - 6)$

Use a difference of two squares pattern to find each product.

5. $(x - 13)(x + 13)$

6. $(p - k)(p + k)$

7. Use the square of a binomial pattern to find the product $(x - 15)^2$.

8. **MATHEMATICAL REFLECTION** Why do the products of $(x - 3)^2$ and $(x + 3)(x - 3)$ have a different number of terms?

Factoring
Factors of Construction

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Vocabulary Organizer, Shared Reading, Quickwrite, Visualization, Think/Pair/Share, Look for a Pattern

A company named Factor Steele Buildings builds metal buildings. They manufacture prefabricated buildings that are customizable. All the buildings come in square or rectangular designs. Most office buildings have an entrance area or Great Room, large offices, and cubicles. The diagram below shows the front face of one of their designs. The distance c represents space available for large offices, and p represents the space available for the great room.

10   10c 10p

c p
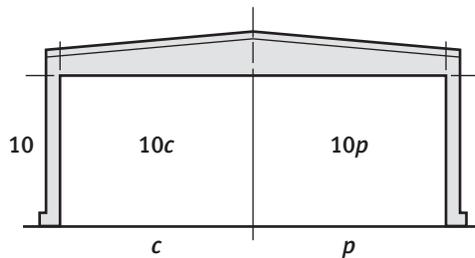
1. To determine how much material is needed to cover the front wall of the building, represent the total area as a product of a monomial and a binomial.

2. Represent the same area from Item 1 as a sum of two monomials.

3. What property can be used to show that the two quantities in Items 1 and 2 are equal?

ACADEMIC VOCABULARY

A **factor** is any of the numbers or symbols that when multiplied together form a product. For example, 2 is a factor of 2x, because 2 can be multiplied by x to get 2x.

To **factor** a number or expression means to write the number or expression as a product of its factors.

My Notes

In Item 2 the number 10 is the **greatest common factor** of the polynomial $10c + 10p$. The **greatest common factor (GCF) of a polynomial** is the greatest monomial that divides into each term of the polynomial without a remainder.

4. Factor Steele Buildings has an expression that they use to input the length of the large office space and it gives the area of an entire space. This expression needs to be simplified. Determine the GCF of the polynomial $6c^2 + 12c - 9$. Explain your choice.

READING MATH

Remember that *factor* can be used as a noun or a verb. See previous page.

To Factor a Monomial (the GCF) from a Polynomial

Steps to Factoring	Example
• Determine the GCF of all terms in the polynomial.	$4x^2 + 2x - 8$ GCF $= 2$
• Write each term as the product of the GCF and another factor.	$2(2x^2) + 2(x) + 2(-4)$
• Use the Distributive Property to factor out the GCF.	$2(2x^2 + x - 4)$

TRY THESE **A**

Factor a monomial from each polynomial.

a. $36x + 9$

b. $6x^4 + 12x^2 - 18x$

c. $15t^3 + 10t^2 - 5t$

d. $125n^6 + 250n^5 + 25n^3$

SUGGESTED LEARNING STRATEGIES: Create Representations,
Graphic Organizer

My Notes

Factor Steele Buildings can create many floor plans with different size spaces. In the diagram below the great room has length and width equaling x units, and each cubicle has a length and width equaling 1 unit. Use the diagram below for Items 5–8.
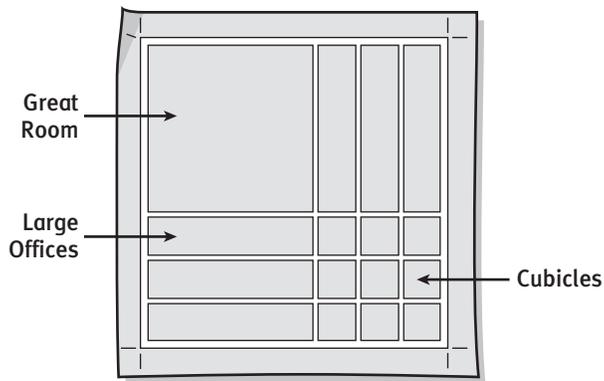
5. Represent the area of the entire office above as a sum of the areas of all the rooms.

6. Write the area of the entire office as a product of two binomials.

7. How are the answers to Items 5 and 6 related? What property can you use to show this relationship?

My Notes

The graphic organizer you used in Tri-Com Computers can be used to represent the office, and help factor a trinomial of the form $x^2 + bx + c$.

8. Complete the graphic organizer below to show the product of $(x + 3)(x + 2)$.
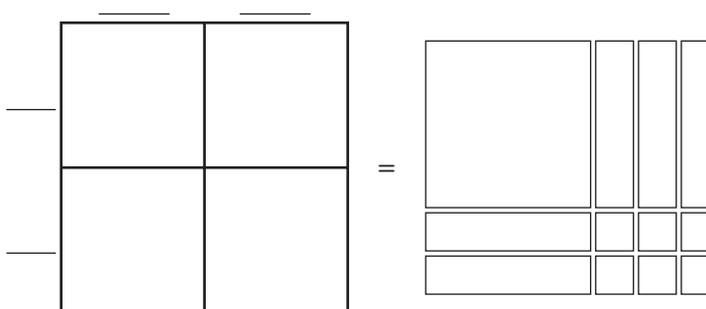
9. Write your answer for Item 8 as a sum of all the regions.

The binomials you multiplied in Item 8 are the factors of the trinomial that represented the entire office in Item 9.

10. Complete the graphic organizer and write the sum in simplified form.

	x^2	$3x$
	$4x$	12
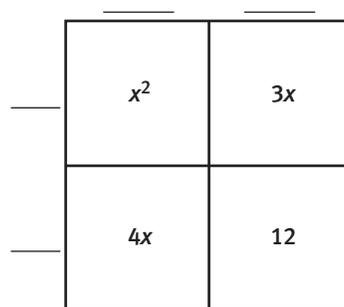
11. Write the binomial factors that were multiplied together to create the graphic organizer in Item 10.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Note Taking

My Notes

12. Use the pattern in the graphic organizer to analyze a trinomial of the form $x^2 + bx + c$. How are the numbers in your binomial factors related to the constant term c, and to b, the coefficient of x?

13. The following steps can be used to factor $x^2 + 12x + 32$. Describe what was done in each step to find the factors of $x^2 + 12x + 32$.

a. **Step 1**

x^2	
	32

b. **Step 2**

32				
32	1	$32 + 1$	$=$	33
16	2	$16 + 2$	$=$	18
8	4	$8 + 4$	$=$	12

c. **Step 3**

	x	8
x	x^2	$8x$
4	$4x$	32

d. **Step 4**

$$x^2 + 12x + 32 = (x + 4)(x + 8)$$

My Notes

14. Fill in the missing sections of the graphic organizer for the trinomial $x^2 - 6x + 8$. Express the trinomial as a product of two binomials.

	_____	_____
_____	x^2	
_____	$-4x$	8

15. Make a graphic organizer like the one above for the trinomial $x^2 + 14x + 45$. Express the trinomial as a product of two binomials.

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Work Backward, Group Discussion, Note Taking

TRY THESE B

Factor each polynomial.

a. $x^2 + 15x + 56$

b. $x^2 + 22x + 120$

c. $x^2 + 6x - 27$

d. $x^2 - 14x + 48$

When attempting to factor trinomials that have a leading coefficient of 1, such as $x^2 + bx + c$, you must focus on the values of b and c.

EXAMPLE

Factor: $x^2 - 6x - 27 = (x \pm \underline{?})(x \pm \underline{?})$.

Step 1: Identify the constant term c. -27

Step 2: Identify b, the coefficient of x. -6

Step 3: Find two numbers whose product is equal to c (-27) and whose sum is equal to b (-6).

Factor 1	Factor 2	Sum
27	−1	26
−27	1	−26
9	−3	6
−9	3	−6

Step 4: Write the binomial factors $(x - 9)(x + 3)$.

TRY THESE C

Factor each trinomial. Write the answer as a product of two binomials.

a. $x^2 + 8x + 15$

b. $x^2 - 5x - 14$

c. $x^2 - 16x + 48$

d. $24 + 10x + x^2$

My Notes

MATH TIP

Divisibility Rules
A number is divisible by

- 2 if the ones digit is divisible by 2.
- 3 if the sum of its digits is divisible by 3.
- 4 if the last two digits are divisible by 4.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the ones digit is 0.

My Notes

16. The polynomial $x^2 - 9$ has two binomial factors. Write the polynomial $x^2 - 9$ as a product of two binomials.

	x	-3
x	x^2	
3		-9
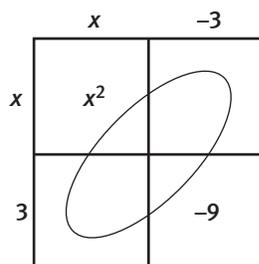
17. Factor each polynomial into two binomial factors:

Polynomial	1st Factor	2nd Factor
$x^2 - 9$		
$x^2 - 16$		
$x^2 - 25$		
$81 - x^2$		
$4x^2 - 49$		
$36x^2 - 4y^2$		

18. Explain how to factor a polynomial of the form $a^2 - b^2$.

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Work Backward, Interactive Word Wall

My Notes

19. Factor each polynomial into two binomial factors.

Polynomial	1st Factor	2nd Factor	First term in each factor	Second term in each factor
$x^2 + 6x + 9$				
$x^2 - 6x + 9$				
$x^2 + 8x + 16$				
$x^2 - 8x + 16$				
$x^2 + 10x + 25$				
$x^2 - 10x + 25$				

20. Explain how to factor polynomials of the form $a^2 + 2ab + b^2$, and $a^2 - 2ab + b^2$.

There are times when polynomials have more than two factors. Use your skills in factoring out monomials and binomials from a polynomial to answer Items 21–23.

21. a. What is the GCF of the polynomial $2x^3 + 16x^2 + 32x$?

   b. Use distributive property to factor out the GCF. Write the polynomial as a product of a monomial and a trinomial.

   c. Factor the trinomial into the product of two binomials. Write the original polynomial as a product of two binomials and a monomial.

My Notes

MATH TERMS

A **prime polynomial** is a polynomial that has no factors other than 1 and itself.

Your answer to Item 21 is *completely factored* when each of its factors is prime. Factoring will be used to help solve equations. A polynomial that is factored in this way will allow a polynomial equation to be solved more readily.

TRY THESE **D**

Completely factor each polynomial.

a. $4x^3 - 64x$

b. $5x^2 - 50x + 80$

c. $4x^3 - 8x^2 - 60x$

d. $2x^3 - 2x^2 - 4x$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Factor the GCF from $4x^3 + 2x^2 + 6x$.

2. Factor the trinomial $x^2 + 11x + 30$.

3. Factor the trinomial $x^2 - 6x + 9$.

4. Factor the trinomial $x^2 + x - 30$.

5. Factor $9x^2 - 144$ completely.

6. Factor $x^2 + 22x + 121$ completely.

7. Factor $3x^3 + 9x^2 + 6x$ completely.

8. **MATHEMATICAL REFLECTION** How does factoring a trinomial relate to the process of multiplying a binomial?

Factoring Trinomials
Factoring by the Letters

LEARNING STRATEGIES: Marking the Text, Close Reading, Summarize/Paraphrase/Retell

You can factor trinomials of the form $ax^2 + bx + c$ with leading coefficient, $a > 0$, in more than one way. In this activity, a, b, and c are integers having no common factors other than one.

Recall that 2×2 boxes were used as graphic organizers to multiply binomials and to factor trinomials of the form $x^2 + bx + c$. A method for factoring a trinomial where $a > 0$ uses a 3×3 box organizer similar to a tic-tac-toe grid.

My Notes

Directions

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

Step 1: Identify the values of a, b, and c from the trinomial. Put a in Box A and c in Box B. Put the product of a and c in Box C.

Step 2: List the factors of the number from Box C and identify the pair whose sum is b. Put the two factors you find in Boxes D and E.

Step 3: Find the greatest common factor of Boxes A and E and put it in Box G.

Step 4: In Box F, place the number you multiply by Box G to get Box A.

Step 5: In Box H, place the number you multiply by Box F to get Box D.

Step 6: In Box I, place the number you multiply by Box G to get Box E.

Solution: The binomial factors whose product gives the trinomial are:
$(Fx + I)(Gx + H)$

A	B	C
F	H	D
G	I	E

EXAMPLE 1

Factor $4x^2 - 4x - 15$.

The factors of -60 will have opposite signs. Since b is negative, the larger factor will be negative. Consider the factors of -60 and identify the pair whose sum is -4. You will put these factors in Boxes D and E in the next step.

| Factor	Factor	Sum		Factor	Factor	Sum
+1	−60	−59		+4	−15	−11
+2	−30	−28		+5	−12	−7
+3	−20	−17		+6	−10	−4

Step 1

A 4	B −15	C −60
F	H	D
G	I	E

MATH TIP

The factors of C will have opposite signs if $C < 0$. If $b < 0$ the factor with the greater absolute value will be negative.

LEARNING STRATEGIES: Predict and Confirm, Discussion Group, Graphic Organizer

My Notes

Step 2				Step 3		
4	−15	−60		4	−15	−60
F	H	−10		F	H	−10
G	I	6		2	I	6

Step 4				Step 5				Step 6		
4	−15	−60		4	−15	−60		4	−15	−60
2	H	−10		2	−5	−10		2	−5	−10
2	I	6		2	I	6		2	3	6
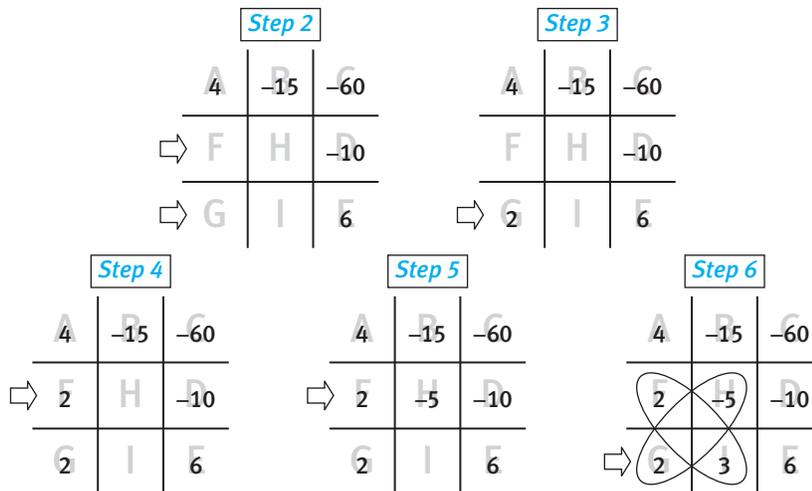
Solution: The trinomial $4x^2 - 4x - 15$ can be factored as $(2x + 3)(2x - 5)$.

Step 1		
A	B	C
F	H	D
G	I	E

Step 2		
A	B	C
F	H	D
G	I	E

TRY THESE A

a. Factor $3x^2 + 4x - 4$.

Fill in the numbers as you follow the directions.

Step 1: Identify $a = $ _____, $b = $ _____, $c = $ _____ Put a in the Box A and c in Box B. Put the product of a and c in Box C.

Step 2: List the factors of the number from Box C and identify the pair whose sum is b. Put the two factors you find in Boxes D and E.

Factor	Factor	Sum		Factor	Factor	Sum

LEARNING STRATEGIES: Look for a Pattern, Predict and Confirm, Guess and Check, Graphic Organizer

My Notes

TRY THESE A (*continued*)

Step 3: Find the greatest common factor of Boxes A and E and put it in Box G.

Step 4: In Box F, place the number you multiply by Box G to get Box A.

Step 5: In Box H, place the number you multiply by Box F to get Box D.

Step 6: In Box I, place the number you multiply by Box G to get Box E.

Step 7: The binomial factors whose product is the given trinomial are $(Fx + I)(Gx + H)$.

Solution: The factors of $3x^2 + 4x - 4$ are _____.

If possible, factor the following trinomials using the graphic organizer.

b. $10x^2 + 19x + 6$ **c.** $6x^2 - 11x - 7$

d. $12x^2 - 11x + 2$ **e.** $5x^2 + 2x - 18$

Step 3–4

A	B	C
F	H	D
G	I	E

Step 5–6

A	B	C
F	H	D
G	I	E
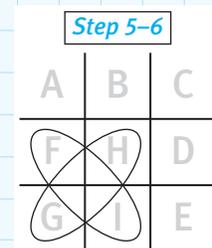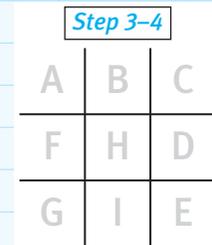
EXAMPLE 2

Factor $2x^2 - 11x + 15$ using a guess and check method.

Possible binomial factors	Reasoning
$(2x\ \ \ )(x\ \ \ )$	$a = 2$. Can be factored as $2 \cdot 1$.
$(2x -\ \ )(x -\ \ )$	$c = 15$. Both factors have the same sign. $b = -11$. Both factors will be negative. 15 can be factored as $1 \cdot 15$, $15 \cdot 1$, $3 \cdot 5$, $5 \cdot 3$
$(2x - 1)(x - 15)$	Product: $2x^2 - 31x + 15$, incorrect
$(2x - 15)(x - 1)$	Product: $2x^2 - 17x + 15$, incorrect
$(2x - 3)(x - 5)$	Product: $2x^2 - 13x + 15$, incorrect
$(2x - 5)(x - 3)$	Product: $2x^2 - 11x + 15$, correct factors

MATH TIP

The factors of c will have the same sign if $c > 0$. If $b < 0$, both factors will be negative. If $b > 0$, both factors will be positive.

Factoring Trinomials
Factoring by the Letters

My Notes

EXAMPLE 3

Factor $3x^2 + 8x - 11$ using a guess and check method.

Possible binomial factors	Reasoning
$(3x\ \ )(x\ \ )$	$a = 3$. Can be factored as $3 \cdot 1$.
$(3x -\ \ )(x +\ \ )$ or $(3x +\ \ )(x -\ \ )$	$c = -11$. The factors have different signs. $b = +8$. The factor with the greater absolute value will be positive. 11 can be factored as $11 \cdot 1, 1 \cdot 11$.
$(3x + 11)(x - 1)$	Product: $3x^2 + 8x - 11$, correct factors
$(3x - 11)(x + 1)$	Product: $3x^2 - 8x - 11x$, incorrect
$(3x - 1)(x + 11)$	Product: $3x^2 + 32x - 11$, incorrect
$(3x + 1)(x - 11)$	Product: $3x^2 - 32x - 11$, incorrect

TRY THESE B

Factor the following trinomials into binomials, if possible.

a. $2x^2 + 15x + 7$ **b.** $3x^2 - 5x - 8$

c. $4x^2 - 27x + 7$ **d.** $6x^2 + 13x - 5$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Factor the following trinomials, if possible.

1. $10x^2 + x - 3$ **2.** $4x^2 + 7x + 3$
3. $3x^2 - 13x + 12$ **4.** $6x^2 - 7x - 5$
5. $7x^2 + 3x - 4$ **6.** $5x^2 - 4x + 3$

7. **MATHEMATICAL REFLECTION** Consider the 3-by-3 graphic organizer used for factoring trinomials. Look at a grid that you used in this lesson. Write about the patterns you see among the nine numbers in the grid and explain why they occur.

Simplifying Rational Expressions
Totally Rational

LEARNING STRATEGIES: Activate Prior Knowledge, Shared Reading, Think/Pair/Share, Note Taking, Group Presentation, Interactive Word Wall

A field trips costs \$800 for the charter bus plus \$10 per student for x students. The cost per student is represented by:

$$\frac{10x + 800}{x}$$

The cost-per-student expression is a **rational expression**. A **rational expression** is the ratio of two polynomials.

Just like fractions, rational expressions can be simplified and combined using the operations of addition, subtraction, multiplication and division.

When a rational expression has a polynomial in the numerator and a monomial in the denominator, it may be possible to simplify the expression by dividing each term of the polynomial by the monomial.

My Notes

ACADEMIC VOCABULARY

rational expression

EXAMPLE 1

Simplify by dividing: $\dfrac{12x^5 + 6x^4 - 9x^3}{3x^2}$

Step 1: Rewrite the rational expression to indicate each term of the numerator divided by the denominator.

$$\frac{12x^5}{3x^2} + \frac{6x^4}{3x^2} - \frac{9x^3}{3x^2}$$

Step 2: Divide. Use the Quotient of Powers Property

$$\frac{12x^5}{3x^2} + \frac{6x^4}{3x^2} - \frac{9x^3}{3x^2}$$

$$4x^{5-2} + 2x^{4-2} - 3x^{3-2}$$

$$4x^3 + 2x^2 - 3x^1$$

Solution: $4x^3 + 2x^2 - 3x$

TRY THESE A

Simplify by dividing.

a. $\dfrac{5y^4 - 10y^3 - 5y^2}{5y^2}$

b. $\dfrac{32n^6 - 24n^4 + 16n^2}{-8n^2}$

My Notes

To simplify a rational expression, first factor the numerator and denominator. Remember that factors can be monomials, binomials or even polynomials. Then, divide out the common factors.

EXAMPLE 2

Simplify $\dfrac{12x^2}{6x^3}$.

Step 1: Factor the numerator and denominator.
$$\frac{2 \cdot 6 \cdot x \cdot x}{6 \cdot x \cdot x \cdot x}$$

Step 2: Divide out the common factors.
$$\frac{2 \cdot \cancel{6} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{6} \cdot x \cdot \cancel{x} \cdot \cancel{x}}$$

Solution: $\dfrac{2}{x}$

EXAMPLE 3

Simplify $\dfrac{2x^2 - 8}{x^2 - 2x - 8}$.

Step 1: Factor the numerator and denominator.
$$\frac{2(x + 2)(x - 2)}{(x + 2)(x - 4)}$$

Step 2: Divide out the common factors.
$$\frac{2\cancel{(x + 2)}(x - 2)}{\cancel{(x + 2)}(x - 4)}$$

Solution: $\dfrac{2(x - 2)}{x - 4}$

MATH TIP

If a, b, and c are polynomials, and b and c do not equal 0, then $\dfrac{ac}{bc} = \dfrac{a}{b}$, because $\dfrac{c}{c} = 1$.

TRY THESE B

Simplify each rational expression.

a. $\dfrac{6x^4y}{15xy^3}$

b. $\dfrac{x^2 + 3x - 4}{x^2 - 16}$

c. $\dfrac{15x^2 - 3x}{25x^2 - 1}$

The value of the denominator in a rational expression cannot be zero, since division by zero is undefined.

- In Example 2, x cannot equal 0 since the denominator $6 \cdot (0)^3 = 0$.
- To find the excluded values of x in Example 3, first factor the denominator. You can see that $x \neq -2$ because that would make the factor $x + 2 = 0$. Also, $x \neq 4$ because that would make the factor $x - 4 = 0$. Therefore, in Example 3, x cannot equal -2 or 4.

LEARNING STRATEGIES: Activating Prior Knowledge, Shared Reading, Note Taking, Group Presentation

To multiply rational expressions first factor the numerator and denominator of each expression. Next, divide out any common factors. Then simplify, if possible.

EXAMPLE 4

Multiply $\dfrac{2x-4}{x^2-1} \cdot \dfrac{3x+3}{x^2-2x}$. Simplify your answer if possible.

Step 1: Factor the numerators and denominators

$$\dfrac{2(x-2)}{(x+1)(x-1)} \cdot \dfrac{3(x+1)}{x(x-2)}$$

Step 2: Divide out common factors.

$$\dfrac{2(x-2) \cdot 3(x+1)}{(x+1)(x-1)(x)(x-2)}$$

Solution: $\dfrac{6}{x(x-1)}$

To divide rational expressions, use the same process as dividing fractions. Write the division as multiplication of the reciprocal. Then solve the multiplication problem.

EXAMPLE 5

Divide $\dfrac{x^2-5x+6}{x^2-9} \div \dfrac{2x-4}{x^2+2x-3}$. Simplify your answer.

Step 1: Rewrite the division as multiplication of the reciprocal.

$$\dfrac{x^2-5x+6}{x^2-9} \cdot \dfrac{x^2+2x-3}{2x-4}$$

Step 2: Factor the numerators and the denominators.

$$\dfrac{(x-2)(x-3)}{(x+3)(x-3)} \cdot \dfrac{(x+3)(x-1)}{2(x-2)}$$

Step 3: Divide out common factors.

$$\dfrac{(x-2)(x-3)(x+3)(x-1)}{(x+3)(x-3)(2)(x-2)}$$

Solution: $\dfrac{x-1}{2}$

MATH TiP

When dividing fractions, you write the division as multiplication of the reciprocal.

$$\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}$$

If a, b, c, and d have any common factors, you can divide them out before you multiply.

$$\dfrac{4}{15} \div \dfrac{8}{3} = \dfrac{4}{15} \cdot \dfrac{3}{8}$$

$$= \dfrac{4}{3 \cdot 5} \cdot \dfrac{3}{2 \cdot 4} = \dfrac{1}{10}$$

TRY THESE C

Multiply or divide. Simplify your answer.

a. $\dfrac{2x+2}{x^2-16} \cdot \dfrac{x^2-5x+4}{4x^2-4}$

b. $\dfrac{3xy}{3x^2-12} \div \dfrac{xy+y}{x^2+3x+2}$

Simplifying Rational Expressions
Totally Rational

My Notes

To add or subtract rational expressions with the same denominator, add or subtract the numerators and then simplify if possible.

EXAMPLE 6

Simplify $\dfrac{10}{x} - \dfrac{5}{x}$.

Step: *Subtract the numerators.* $\dfrac{10}{x} - \dfrac{5}{x} = \dfrac{10-5}{x}$

Solution: $\dfrac{5}{x}$

EXAMPLE 7

Simplify $\dfrac{2x}{x+1} + \dfrac{2}{x+1}$.

Step 1:   *Add the numerators.* $\dfrac{2x}{x+1} + \dfrac{2}{x+1} = \dfrac{2x+2}{x+1}$

Step 2:   *Factor.* $= \dfrac{2(x+1)}{x+1}$

Step 3:   *Divide out common factors.* $= \dfrac{2(\cancel{x+1})}{\cancel{x+1}}$

Solution: 2

TRY THESE D

Add or subtract. Simplify your answer.

a. $\dfrac{3}{x^2} - \dfrac{x}{x^2}$

b. $\dfrac{2}{x+3} - \dfrac{6}{x+3} + \dfrac{x}{x+3}$

c. $\dfrac{x}{x^2-x} + \dfrac{4x}{x^2-x}$

LEARNING STRATEGIES: Activating Prior Knowledge, Note Taking, Graphic Organizer, Group Presentation, Identify a Subtask

To add or subtract rational expressions with unlike denominators, find a common denominator. The least common multiple (LCM) of the denominators is used for the common denominator.

The easiest way to find the LCM is to factor each expression. The LCM is the product of each factor common to the expressions as well as any non-common factors.

MATH TERMS

The **least common multiple** is the smallest multiple that two or more numbers have in common.

The numbers 10 and 25 have many common multiples. The number 50 is the least common multiple.

EXAMPLE 8

Find the LCM of $x^2 - 4$ and $2x + 4$.

Step 1: *Factor each expression.*

$$x^2 - 4 = (x + 2)(x - 2)$$
$$2x + 4 = 2(x + 2)$$

Step 2: *Identify the factors.*

Common Factor: $(x + 2)$

Factors Not in Common: 2 and $(x - 2)$

Step 3: *The LCM is the product of the factors in Step 2.*

Solution: LCM: $2(x + 2)(x - 2)$

TRY THESE **E**

a. Find the LCM of $2x + 2$ and $x^2 + x$.

Factor each expression:

_____

_____

Factors in Common: _____

Factors Not in Common: _____

LCM: _____

b. Find the LCM of $x^2 - 2x - 15$ and $3x + 9$.

LEARNING STRATEGIES: Activating Prior Knowledge, Think/Pair/Share, Note Taking

My Notes

> **MATH TIP**
>
> Multiplying a fraction by a form of 1 gives an equivalent fraction.
> $\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$, because $\frac{2}{2} = 1$.
> The same is true for rational expressions. Multiplying by $\frac{(x-2)}{(x-2)}$ gives an equivalent expression because $\frac{(x-2)}{(x-2)} = 1$.

Now you are ready to add and subtract rational expressions with different denominators. First find the LCM of the denominators. Next write each fraction with the LCM as the denominator. Then add or subtract. Simplify if possible.

EXAMPLE 9

Subtract $\frac{2}{x} - \frac{3}{x^2 - 2x}$. Simplify your answer if possible.

Step 1: Find the LCM.

Factor the denominators. x
$x(x-2)$

The LCM is $x(x-2)$

Step 2: Multiply the numerator and denominator of the first term by $(x-2)$. The denominator of the second term is the LCM. $\frac{2}{x} \cdot \frac{(x-2)}{(x-2)} - \frac{3}{x(x-2)}$

Step 3: Use the distributive property in the numerator. $\frac{2x-4}{x(x-2)} - \frac{3}{x(x-2)}$

Step 4: Subtract the numerators. $\frac{2x-7}{x(x-2)}$

Solution: $\frac{2x-7}{x(x-2)}$

EXAMPLE 10

Add $\frac{-4}{5-p} + \frac{3}{p-5}$. Simplify your answer if possible.

Step 1: Find a common denominator. $p-5$

> **MATH TIP**
>
> If you multiply $(5-p)$ by -1, you will get $p-5$.

Step 2: Multiply the numerator and denominator of the first term by -1. $\frac{-4}{5-p} \cdot \frac{-1}{-1} + \frac{3}{p-5}$

Step 3: Multiply. $\frac{4}{p-5} + \frac{3}{p-5}$

Step 4: Add. $\frac{7}{p-5}$

Solution: $\frac{7}{p-5}$

Simplifying Rational Expressions
Totally Rational

LEARNING STRATEGIES: Graphic Organizer, Group Presentation

TRY THESE **F**

a. Add $\dfrac{1}{x^2 - 1} + \dfrac{2}{x + 1}$. Directions are provided to help you.

Factor each denominator:

_____

_____

Common Factors: _____

Factors Not in Common: _____

LCM: _____

Fill in the blanks on the left and write the algebraic steps on the right.

Factor the denominator of the first term. Multiply the numerator and denominator of the second term by _____. Add the numerators. Use the distributive property. Combine like terms. Solution:	$\dfrac{1}{x^2 - 1} + \dfrac{2}{x + 1}$

LEARNING STRATEGIES: Group Presentation

My Notes

TRY THESE **F** (continued)

Add or subtract. Simplify your answer.

b. $\dfrac{3}{x+1} - \dfrac{x}{x-1}$

c. $\dfrac{2}{x} - \dfrac{3}{x^2 - 3x}$

d. $\dfrac{2}{x^2 - 4} + \dfrac{x}{x^2 + 4x + 4}$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Simplify by dividing.

1. $\dfrac{16x^5 - 8x^3 + 4x^2}{4x^2}$

Simplify.

2. $\dfrac{3x^2yz}{12xyz^3}$ **3.** $\dfrac{x^2 - 2x + 1}{x^2 + 3x - 4}$ **4.** $\dfrac{2x^2}{4x^3 - 16x}$

Perform the indicated operation.

5. $\dfrac{x^2 - 5x - 6}{x^2 - 4} \cdot \dfrac{x + 2}{x^2 - 12x + 36}$

6. $\dfrac{x^3}{x^2 - 1} \cdot \dfrac{2x + 2}{4x}$

7. $\dfrac{x^2 + 4x + 4}{2x + 4} \div \dfrac{x^2 - 4}{x^2 - 5x + 6}$

8. $\dfrac{1}{x-1} \div \dfrac{x}{x-1}$

Find the least common multiple.

9. $2x + 4$ and $x^2 - 4$

10. $x + 3$, $x^2 + 6x + 9$, and $x^2 - 7x - 30$

Perform the indicated operation.

11. $\dfrac{x}{x+1} - \dfrac{2}{x+3}$

12. $\dfrac{2}{3x - 3} - \dfrac{x}{x^2 - 1}$

13. **MATHEMATICAL REFLECTION** What have you learned about simplifying rational expressions as a result of this activity?

Polynomial Operations and Factoring

MEASURING UP

When you apply for a job at Ship-It-Quik you have to perform computations involving volume and surface area. For a rectangular prism, the volume is $V = lwh$ and surface area is $SA = 2lw + 2wh + 2lh$ where l, w, and h are length, width and height, respectively.

1. In the first part of the job application, you have to verify whether or not the following computations are correct. Explain your reasoning by showing your work.

Volume:
$2x^2 + 15x + 36$

Surface Area:
$10x^2 + 90x + 72$
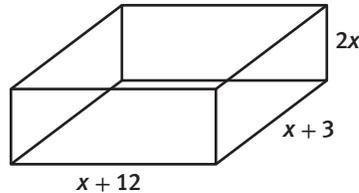
2. For the second part of the job application, you are given a box whose volume is $3x^3 + 21x^2 + 30x$. Two of the dimensions are shown, $3x$ and $x + 2$. What is the other dimension?
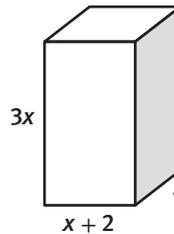
3. For the final part of the job application, find the volume of a cylinder with radius $3x^2y$ and height $2xy$. Use the formula $V = \pi r^2 h$ where r is the radius and h is the height. Simplify your answer as much as possible.

Polynomial Operations and Factoring

MEASURING UP

	Exemplary	Proficient	Emerging
Math Knowledge #1, 2, 3	The student: • Correctly verifies whether the computations are correct for both the volume and the surface area. (1) • Finds the correct third dimension. (2) • Correctly uses the formula to find and simplify the expression for the volume of the cylinder. (3)	The student: • Attempts to verify the validity of the computations for both attributes, but only one is correct. • Uses the correct method to find the third dimension but makes a mathematical error. • Correctly substitutes in the formula but makes a mathematical error when simplifying it.	The student: • Attempts to verify the validity of the computations for both attributes, but neither is correct. • Attempts but is unable to find the third dimension. • Substitutes in the formula incorrectly.
Communication #1	The student correctly justifies his/her verification of both answers given. (1)	The student correctly justifies his/her verification of only one of the answers given; the second justification is incomplete but contains no mathematical errors.	The student attempts to justify his/her verification, but the answer contains mathematical errors.

ACTIVITY 4.1

1. $\frac{x^2}{5x} =$

   a. x

   b. $\frac{x}{5}$

   c. $5x$

   d. $\frac{5}{x}$

2. $3x^{-2}y^3 \cdot 5xy =$

   a. $\frac{8y^3}{x^2}$

   b. $\frac{15y^4}{x}$

   c. $3 - x$

   d. $\frac{3x + 3}{x}$

3. $-4x^2(-3xy^4)^2 =$

   a. $-24x^4y^6$

   b. $48x^4y^6$

   c. $24x^4y^8$

   d. $-36x^4y^8$

4. $\left(\frac{4}{x^3}\right)^{-3} =$

   a. $-12x^6$

   b. $-12x^9$

   c. $\frac{x^9}{64}$

   d. $\frac{64}{x^6}$

5. Mars is about 1.2×10^8 miles from Earth. If the Mars Polar Lander is sending messages back to earth at about 1.86×10^5 miles per second (the speed of light), how long will it take for the data to arrive?

ACTIVITY 4.2

Graph each of the following functions. Choose an appropriate scale for your coordinate grid.

6. $y = (2.5)^x$

7. $y = (0.75)^x$

8. $y = 3(1.5)^x$

9. $y = 8(0.25)^x$

For each of the following descriptions of exponential functions, write the function.

10. Initial value is 50. Constant ratio is 3.5.

11. Initial value is 100. Constant ratio is 0.9.

12. Initial value is 10. Constant ratio is 0.25.

13. Initial value is $\frac{1}{3}$. Constant ratio is 3.

ACTIVITY 4.3

Express each expression in simplest radical form.

14. $\sqrt{56}$

15. $\sqrt{324}$

16. $\sqrt{12x^2}$

17. $\sqrt{25a^3b^4}$

18. $5\sqrt{15} - 2\sqrt{15}$

19. $14\sqrt{6} + 3\sqrt{2} - 2\sqrt{8}$

20. $5 - 3\sqrt{7}$

21. $2\sqrt{63} + 6\sqrt{28} + 8\sqrt{45}$

22. $(\sqrt{20})(\sqrt{10})$

23. $\left(\sqrt{\frac{2}{3}}\right)\left(\sqrt{\frac{3}{2}}\right)$

24. $(3\sqrt{3})(4\sqrt{18} + \sqrt{6})$

25. $\dfrac{\sqrt{3}}{\sqrt{5}}$

26. $\left(\sqrt{\dfrac{1}{3}}\right)\left(\sqrt{\dfrac{1}{12}}\right)$

27. $\sqrt{\dfrac{4}{7}}$

28. When given the surface area, *SA*, of a cube the length of an edge, *e*, can be found using the formula $e = \sqrt{\dfrac{SA}{6}}$. Find the edge length of cubes with these surface areas.

  a. 24 in.² and 96 in.²

  b. 54 in.² and 486 in.²

  c. Compare and contrast the surface areas and edge lengths for each pair of cubes in parts (a) and (b).

  d. If the surface area of a cube is 25 times greater than the surface area of another cube, how do you think their edges compare? Explain your reasoning.

ACTIVITY 4.4

For Items 1–5, use the polynomial
$3x^5 - \dfrac{7}{8}x^3 + 13x + \dfrac{4}{3}.$

29. Name the coefficients of the polynomial.

30. List the terms, and specify the degree of each term.

31. What is the degree of the polynomial?

32. What is the leading coefficient of the polynomial?

33. What is the constant term of the polynomial?

Add or subtract.

34. $(9x^2 + 3x + 5) + (x^4 + x^2 - 12x - 4)$

35. $(4x^3 + 9x - 22) - (8x^3 + 3x^2 - 7x + 11)$

36. $\left(\dfrac{2}{3}x^2 + \dfrac{1}{5}x + \dfrac{5}{8}\right) + \left(-\dfrac{1}{2}x^3 + \dfrac{4}{3}x^2 - \dfrac{3}{5}x - \dfrac{3}{8}\right)$

37. $\left(\dfrac{3}{4}x^3 + 4x - \dfrac{5}{6}\right) - \left(-\dfrac{1}{2}x^3 + 2x^2 - 5x + \dfrac{2}{3}\right)$

ACTIVITY 4.5

Find the product.

38. $\left(\dfrac{1}{4} + \dfrac{2}{3}\right)^2$

39. $(4 + 6)(15 + 8)$

40. $(x - 11)(x - 13)$

41. $\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{3}{7}\right)$

42. $(2x - 7)(2x + 7)$

43. $(3w - 2y)(3w + 2y)$

44. $(6x - 8)^2$

45. $(x - 4)(2x^2 - 5x - 10)$

ACTIVITY 4.6

46. Find the GCF of: $34x^6 + 17x^4 + 51x^2$.

Factor each polynomial.

47. $x^2 + 20x + 51$

48. $x^2 - 5x + 6$

49. $x^2 - 22x + 72$

50. $16x^2 - 169$

51. $x^2 + 8x + 16$

52. $5x^3 + 45x^2 + 40x$

ACTIVITY 4.7

Factor the trinomials, if possible.

53. $6x^2 + 11x + 4$

54. $9x^2 - 34x - 8$

55. $8x^2 + 14x - 15$

56. $10x^2 - 13x + 3$

57. $3x^2 + x - 4$

58. $2x^2 - 9x + 10$

59. $6x^2 + 7x - 2$

60. $5x^2 - 7x + 2$

ACTIVITY 4.8

61. Simplify by dividing. $\dfrac{35a^7 + 15a^5 - 10a^3}{5a^3}$

a. $7a^4 + 3a^2 - 2$

b. $7a^4 + 3a^2 - 2a$

c. $7a^4 + 3a^2 + 2$

d. $7a^7 + 3a^5 - 2a^3$

62. Simplify $\dfrac{x^2 - 25}{5x + 25}$.

a. $x - 5$

b. $\dfrac{x}{5} + 1$

c. $\dfrac{x - 5}{5}$

d. $\dfrac{x - 5}{x + 5}$

63. $\dfrac{3x + 3}{x^2} \cdot \dfrac{x^2 - x}{x^2 - 1} =$

a. 3

b. $\dfrac{3}{x}$

c. $3 - x$

d. $\dfrac{3x + 3}{x}$

64. $\dfrac{x^2 - 10x + 24}{x^2 - 36} \div \dfrac{5x - 20}{x^2 + 3x - 18}$

a. $x - 3$

b. $\dfrac{x - 3}{5}$

c. $\dfrac{x + 6}{5}$

d. $\dfrac{x - 4}{5(x - 3)}$

65. Find the LCM of $x^2 - 6x + 9$ and $4x - 12$.

a. $(x - 3)$

b. $4(x - 3)$

c. $(x - 3)(x + 3)$

d. $4(x - 3)^2$

66. $\dfrac{x}{x - 6} + \dfrac{x - 12}{x - 6} =$

a. 2

b. 4

c. $\dfrac{12}{x - 6}$

d. $\dfrac{x^2 - 12x}{(x - 6)^2}$

67. $\dfrac{x}{x - 1} - \dfrac{1}{x} =$

a. 1

b. $\dfrac{1}{x}$

c. $1 - x$

d. $\dfrac{x^2 - x + 1}{x^2 - x}$

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

   - How do multiplicative patterns model the physical world?
   - How are adding and multiplying polynomial expressions different from each other?

Academic Vocabulary

2. Look at the following academic vocabulary words:
   - coefficient
   - degree of a polynomial
   - difference of two squares
   - factor
   - polynomial
   - radical expression
   - rational expression
   - term

Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

   a. What will you do to address each weakness?

   b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

1. Which expression is equal to $(-3xy^2)^3$?

   A. $-9x^3y^6$

   B. $-9x^3y^5$

   C. $-27x^3y^6$

   D. $-27x^3y^5$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. What is the value of x, if the value of $\sqrt{2x}$ is 4?

2.

3. What is the value of x in the equation $\dfrac{x-2}{3} = \dfrac{x+1}{4}$?

3.

Read
Solve
Explain

4. The current I that flows through an electrical appliance is determined by $I = \sqrt{\dfrac{P}{R}}$, where P is the power required and R is the resistance of the appliance. The current is measured in amperes, the power in watts, and the resistance in ohms. A hair dryer has a resistance of 8 ohms and draws 15 amperes of current. How much power does it use? Show your work.

Answer and Explain

_____

_____

_____

_____

_____